

$$(2) \begin{cases} x^3 + y^3 + (x+y)xy = 13 \\ \frac{x^3 y^3}{x+y} = 36. \end{cases}$$

2. Find the number of variations of n different letters taken r together; also the number of such variations, when each may enter 1, 2, 3, etc., or r times in each variation.

If the number of variations of $a+b$ things taken two together be 56, and of $a-b$ things 12, find the number of combinations of a things, taken b together.

3. State the Binomial Theorem, and prove it when the index is a positive integer.

Expand to five terms, $(a-3x)^{-\frac{1}{2}}$.

4. Find the present value of an annuity A for n years at compound interest.

The reversion of a freehold estate worth P pounds per annum to commence a years hence is to be sold. Ascertain its present value at R per cent. per annum compound interest.

5. Define a *continued fraction*, and illustrate the method of converting a quadratic surd to a continued fraction.

Express as continued fractions

$$(1) \sqrt{11}; (2) \sqrt{13}; (3) \sqrt{17}.$$

6. What is a recurring series?

Explain what is meant by the *scale of relation* of a recurring series.

Sum to n terms, and *ad infinitum* the series

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots$$

7. Find the radii of the inscribed and escribed circles of a triangle in terms of the sides and angles.

8. In any triangle prove:

$$(1) \frac{\sin(B-C)}{\sin(C-A)} = \frac{(b^3 - c^3)}{(c^3 - a^3)} \frac{\sin B}{\sin C}.$$

$$(2) \text{Area} = \frac{1}{2} (b^3 + c^3) \frac{a \sin B \sin C}{b \sin B + c \sin C}.$$

9. Show how to expand a^x in a series of ascending powers of x .

10. State Demoivre's Theorem, and assuming its truth, prove,

$$(1) \cos a = 1 - \frac{a^2}{1.2} + \frac{a^4}{1.2.3.4} \dots \text{etc.}$$

$$(2) \sin a = a - \frac{a^3}{1.2.3} + \frac{a^5}{5!} \dots$$

11. Sum to n terms:

$$\sin \theta - \sin(\theta + a) + \sin(\theta + 2a) \dots$$

and deduce the sum of n terms of the series $\cos \theta - \cos 2\theta + \cos 3\theta \dots$ etc.

First Examination (Pass.)

$$1. (1) \text{ Given } \begin{cases} x:y::a:b \\ x^2 + y^2 = c^2 \end{cases}$$

find the values of x and y .

$$(2) \text{ Given } \begin{cases} 2x + 4y - 3z = 22 \\ 4x - 2y + 5z = 18 \\ 6x + 3y - 2z = 31 \end{cases}$$

find the values of x , y , and z .

2. Solve the following equations:

$$(1) \begin{cases} x^2 + y^2 = 41 \\ xy = 20 \end{cases}$$

$$(2) x^4 - 4x^3 + 6x^2 - 4x - 15 = 0.$$

$$(3) \begin{cases} x^3 + xy + y^3 = 7 \\ x^4 + x^2y^2 + y^4 = 21 \end{cases}$$

3. Define an arithmetical and a geometrical series.

(1) Find the n^{th} term, and the sum of n terms of an arithmetical series.

(2) Insert five arithmetical means between 3 and 16.

4. In a geometrical series, if the ratio be a proper fraction, show that the sum of the series when the number of terms is increased indefinitely has a limiting value.

The limit of the sum of a geometrical series is $3\frac{1}{2}$, and the second term is $-\frac{5}{2}$; find the series.

5. Find three numbers in geometrical progression such that their sum shall be 21, and the sum of their squares 189.

6. Define the trigonometrical ratios of an angle less than 90° , and prove:

$$(1) \sin^2 A + \cos^2 A = 1.$$

$$(2) \sin A \cos A = \frac{1}{\tan A + \cot A}.$$

7. Prove the following formulæ:

$$(1) \sin A - B = \sin A \cos B - \cos A \sin B.$$

$$(2) \tan \frac{1}{2} A = \frac{1 - \cos A}{\sin A}.$$