EF is the common tangent; shew that twice the square on EF is equal to the rectangle $ABCD$.

14. Draw a straight line, so that the part of it intercepted between one side of a given isosceles triangle and the other side produced, shall be equal to a given line, and be bisected by the base.

FIRST-CLASS TEACHERS.—GRADE C.

ALGEBRA—JULY, 1880.

1. If in $ax^2+2bxy+cy^2$, $ku+lv$ be substituted for x and $mu+nv$ for y, the result takes the form

$$Au^2+2Buv+Cv^2$$

find the value of $(B^2-AC) \div (b^2-ac)$ in terms of k, l, m, n.

2. Resolve into factors

$$a(b-c)^3+b(c-a)^3+c(a-b)^3.$$

Prove that

$$\frac{Au^3+Bv^3+Cw^3}{uvw}=\frac{Ax^3+By^3+Cz^3}{xyz}$$

if $u=x(By^3-Cz^3)$
$v=y(Cz^3-Ax^3)$
$w=z(Ax^3-By^3)$

3. Extract the square root of

$$(a-b)^2(b-c)^2+(b-c)^2(c-a)^2+(c-a)^2(a-b)^2$$

and the cube root of

$$4\{(a-b)^6+(b-c)^6+(c-a)^6 - 3(a-b)^2(b-c)^2(c-a)^2\}$$

4. Eliminate x, y, z from

$$ax+by+cz=1, \frac{a}{x}=\frac{b}{y}=\frac{c}{z},$$

$$K(x^2+y^2+z^2)+2(lx+my+nz)+h=0$$

5. Simplify $\dfrac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}+\sqrt{b}}$

$$\{\sqrt{(4+3i)}+\sqrt{(4-3i)}\}^2$$

and $\left(\dfrac{-1+i\sqrt{3}}{2}\right)^2+\dfrac{-1+i\sqrt{3}}{2}+1$

in which i is $\sqrt{-1}$.

6. Given the first term, the common difference, and the number of terms in an arithmetical progression; find (i) the sum of the terms, (ii) the sum of the squares of the terms.

7. Solve the equations

(i) $(a-x)^3=(x-b)^3$

(ii) $ax+by=\dfrac{a}{x}+\dfrac{b}{y}=1$

(iii) $x(y+z^{-1})=a$
$y(z+x^{-1})=b$
$z(x+y^{-1})=c$

8. What value (other than 1) must be given to q, that one of the roots of $x^2-2x+q=0$ may be the square of the other?

If a, b, c are the roots of $x^3-px^2+qx-r=0$, express

$$\frac{2a^2b^2+2b^2c^2+2c^2a^2-a^4-b^4-c^4}{2ab+2bc+2ca-a^2-b^2-c^2}$$

in terms of p, q, r.

9. A vessel makes two runs on a measured mile; one with the tide in m minutes and the other against the tide in n minutes. Find the speed of the vessel through the water, and the rate the tide was running at, assuming both to be uniform.

10. Five points A, B, C, O and P lie on a right line. The distances of A, B and C, measured from the point O, are a, b, c; their distances measured from the point P are x, y, and z. Prove that whatever be the position of the points O and P,

$$x^2(b-c)+y^2(c-a)+z^2(a-b)$$
$$+(a-b)(b-c)(c-a)=0.$$