(1) $a^3-3a^2b+3ab^2-b^3+a^2x-b^2x$.
(2) $x^4-(a^2-b-c)x^2-a(b-c)x+bc$.
(3) $(a-b)^6-b^6$.

2. Examine in what cases $a^n \pm b^n$ is divisible by $a \pm b$.

3. Prove the rule for finding the L. C. M. of two algebraic quantities.

Find that of
$x^2-9y^2,\ x^3+3x^2y+4xy^2+12y^3,\ x^3-3x^2y+4xy^2-12y^3$.

4. Simplify

(1) $$\frac{yz}{x(x^2-y^2)(x^2-z_2)}+\frac{zx}{y(y^2-z^2)(y^2-x^2)}+\frac{xy}{z(z^2-x^2)(x^2-y^2)}$$

(2) $$\frac{ab+bc}{ca}\left(\frac{1}{bc}-\frac{1}{ab}\right)+\frac{bc+ca}{ab}\left(\frac{1}{ca}-\frac{1}{bc}\right)+\frac{ca+ab}{bc}\left(\frac{1}{ab}-\frac{1}{ca}\right).$$

5. If $x^5+px^4+qx^2+px+1$ be divisible by x^3+px^2+px+1, then $p+1=q$.

6. (1) If $y+z=ax$, $z+x=by$, $x+y=cz$, then
$$\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=1.$$

(2) If $x+y+z=0$, then
$x^2(y+z)+y^2(z+x)+z^2(x+y)+3xyz=0$.

7. Show that $x^2+y^2>2xy$.

Prove
(1) $a^3+b^3>a^2b+b^2a$, if the greater of a and b be positive.
(2) $(a+b-c)^2+(a+c-b)^2+(b+c-a)^2>ab+bc+ca$.

8. Assuming that $a^m \times a^n=a^{m+n}$ for all values of m and n, shew that
$$a^{\frac{p}{q}}=\sqrt[q]{a^p}=\left(\sqrt[q]{a}\right)^p.$$

9. Shew how to extract the square root of $a+\sqrt{b}$.

Simplify $$\frac{2+\sqrt{2}}{\sqrt{2}+\sqrt{3-2\sqrt{2}}}+\frac{2-\sqrt{2}}{\sqrt{2}-\sqrt{3+2\sqrt{2}}}.$$

Rationalize the denominator of
$$\frac{1}{1+\sqrt{2}+\sqrt{3}}.$$

II.

1. If $x^2+y^2+z^2+2xyz=1$, shew that

(1) $\{(1-y^2)(1-z^2)\}^{\frac{1}{2}}+\{(1-z^2)(1-x^2)\}^{\frac{1}{2}}+\{(1-x^2)(1-y^2)\}^{\frac{1}{2}}=z(1+x)+x(1+y)+y(1+z)$.

(2) $$\left\{\frac{1-x}{1+x}\cdot\frac{1-y}{1+y}\right\}^{\frac{1}{2}}+\left\{\frac{1-y}{1+y}\cdot\frac{1-z}{1+z}\right\}^{\frac{1}{2}}+\left\{\frac{1-z}{1+z}\cdot\frac{1-x}{1+x}\right\}^{\frac{1}{2}}=1.$$

2. Solve the equations
(1) $zx+xy-yz=x^2-a^2,\ xy+yz-zx=y^2-b^2,\ yz+zx-xy=z^2-c^2$.
(2) $(x+y)a^2=x,\ (x+y)b^2=y$; explain the results.

3. Sum the following series:
(1) $1^2+2^2+3^2+$......to n terms.
(2) $\frac{2}{3}+\frac{4}{3^2}+\frac{6}{3^3}+\frac{8}{3^4}+$ ...... to infinity if convergent; to n terms if not convergent.

4. (1) Find the whole number of permutations of n things when each may occur once, twice, thrice......up to r times.

(2) Find the sum of the different numbers that can be formed with m digits a, n digits b, &c., the entire series of $m+n+$..... digits being used in forming each number.

5. If the Binomial Theorem holds for a positive integer, shew that it holds for a positive fraction.

Shew that
$$\left\{\frac{x}{x-1}\right\}^{\frac{1}{x}}=1+\frac{1}{x^2}+\frac{x+1}{\lfloor 2}\cdot\frac{1}{x^4}+\frac{(x+1)(2x+1)}{\lfloor 3}\cdot\frac{1}{x^6}+\ldots\ldots$$

6. (1) As a problem in combinations, without reference to multinomial theorem formulæ, find the coefficient of $a^nb^nc^n$ in the expansion of $(a+b+c)^{3n}$, n being a positive integer.

(2) $\overset{n}{C}_r=\dfrac{n.\overline{n-1}\ldots\ldots\overline{n-r+1}}{\lfloor r}$, then the coeff. of the middle term of $(1+x+x^2)^n$ is

$$1+\overset{n}{C}_1\overset{n-1}{C}_1+\overset{n}{C}_2\overset{n-2}{C}_2+\ldots\ldots\overset{n}{C}_{\frac{n}{2}}\overset{\frac{n}{2}}{C}_{\frac{n}{2}},\text{ or}$$

$$1+\overset{n}{C}_1\overset{n-1}{C}_1+\overset{n}{C}_2\overset{n-2}{C}_2\ldots\ldots\overset{n}{C}_{\frac{n-1}{2}}\overset{\frac{n+1}{2}}{C}_{\frac{n-1}{2}}.$$

7. Shew that $e^x=1+x+\dfrac{x^2}{\lfloor 2}+\ldots\ldots$

If in the equation
$$a^x=x$$
x be a small quantity whose powers above the second may be neglected, shew how to find x approximately.

8. Examine in which cases the series
$$\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\ldots\ldots$$
is convergent or divergent.

If the series be convergent it is greater than
$$\frac{1}{2}\cdot\frac{2^p-1}{2^{p-1}-1},\text{ but less than }\frac{2^{p-1}}{2^{p-1}-1}.$$

9. (1) Every convergent is nearer to the continued fraction than any of the preceding convergents.

(2) Any convergent is nearer to the continued fraction than any other fraction which has a smaller denominator than the convergent has.

(3) The ratio between the area of a regular decagon described about a circle and that of another within the circle is
$$\frac{3}{7}+\frac{1}{4}+\frac{1}{4}+\ldots\ldots$$

SOLUTIONS OF ALGEBRA EXERCISE IN DECEMBER ISSUE.

1. Left side $=\frac{s}{a}-1+\frac{s}{b}-1+\ldots\ldots=\ldots\ldots$

2. Left side $=1-\frac{a}{s}+1-\frac{b}{s}+\ldots\ldots$
$$=n-\frac{a+b+\ldots\ldots}{s}=n-1.$$

3. It should be $s=a+b+c$. Then
$s(s-a)(s-b)(s-c)=2a^2b^2+2b^2c^2+2c^2a^2-a^4-b^4-c^4$
$=4(xy+yz+zx)$.

4. (1) Evidently $a+b+c=x+y+z$,
and $2(b+c)=2x+y+z$; $\therefore b+c-a=x$, &c.
By preceding question $a^4+\ldots\ldots$
$=-(a+b+c)(b+c-a)(c+a-b)(a+b-c)=-(x+y+z)xyz$.
(2) $8abc=(x+y)(y+z)(z+x)=(x+y+z)(xy+yz+zx)-xyz$.

5. $x^2-yz=a^2$; $\therefore x^3+xyz=a^2x$, &c.
$$\therefore \frac{a^2x+b^2y+c^2z}{x+y+z}=\frac{x^3+y^3+z^3-3xyz}{x+y+z}$$
$x^2+y^2+z^2-xy-yz-zx=a^2+b^2+c^2$.

6. $xyz+x+y+z$
$$=\frac{1}{abc}\{(b-c)(c-a)(a-b)+bc(b-c)+ca(c-a)+ab(a-b)\}=0.$$