derived their knowledge of algebra from the Arabians; and either they or the Spaniards were the first to introduce the study into Europe. Strachey, an Oriental scholar of the beginning of this century, furnishes an analysis of an Arabian work in which the ordinary index law is set forth, and methods are furnished for the solution of various forms of simple and quadratic equations, methods identical with those we employ at present.

Leonardo, a merchant of Pisa, acquired while in the East a knowledge of the art, and about 1202 wrote a treatise. His manuscript was never printed, though it is described as "orderly and regular, teaching and demonstrating all the rules, and illustrating them with many examples." In this no notation such as we use was employed; both the quantities and the several operations were expressed by their names, or words at full length. Diophantine problems are treated of, and equations as far as quadratics. His solutions of quadratics are based on geometrical considerations, and it may satisfy the curiosity of some to give the following: To solve $x^2 + ax = n$; here $x = \sqrt{(\frac{1}{4}a^2 + n)} - \frac{1}{2}a$. For take any straight line AB greater than $\frac{1}{2}a$, and on it describe the square $ABDE$. From BA, BD cut off BC, BF each equal to $\frac{1}{2}a$: through C and F draw CGK and FGH parallel to AE and AB respectively. Let AC denote the required quantity x. Then CF will be $\frac{1}{4}a^2$, HK will be x^2, and AG, GD will each be $\frac{1}{2}ax$. Hence the whole square $ABDE$ will be $x^2 + ax + \frac{1}{4}a^2$. But $x^2 + ax = n$. Therefore $ABDE = n + \frac{1}{4}a^2$; and its side AB will be $\sqrt{n + \frac{1}{4}a^2}$; and $x = AB - BC = \sqrt{n + \frac{1}{4}a^2} - \frac{1}{2}a$.

The first printed treatise on Algebra was that of Lucas de Burgo, a Franciscan, who seems to have been instructed in the science in both Italy and the East. His first work was printed about 1470. He gives names to the various powers of the unknown. Thus *co.* or *cosa* is the thing or first power of the unknown; *ce.* or *censo* the product or square, &c., and *n°.* or *numero*, the known number; *p* stands for *piu* or *plus*, *m* for *meno* or minus. Thus where we would put $8 - 4x + 7x^2$, he would write *3n°. m.4co. p.7ce.* He treats of the solution of simple and quadratic equations, of higher equations that may be resolved into quadratics, of surds, their addition, subtraction, multiplication, and division, and of the extraction of the square root of binomial surds.

These facts exhibit the state of algebra among Europeans about the beginning of the fifteenth century. They could solve simple and quadratic equations, using only positive roots, and one unknown; their marks and signs were only abbreviations of words, or the words themselves; and they confined themselves to resolving certain numerical problems. The next great advance is that with which the name of Cardan is usually associated, and consisted in the solution of cubic equations. The circumstances attending the discovery of this solution are amongst the most curious in the romance of science. About 1508 Ferrei, a professor of mathematics at Bologna, had devised a means of solving a particular class of these equations, and, jealous of his discovery, had communicated it to but a few even of his own pupils. One of these, Florido, vain of his knowledge, challenged Tartalea of Brescia to a contest, in which each was to propose to the other thirty questions, and he who first effected the solution of his opponent's problems should win thirty treats for himself and friends. Tartalea appears to have possessed no inconsiderable mathematical power, for he completely defeated Florido, extending his solution of cubics to classes which neither he nor his master had been able to resolve. Cardan at this time was a physician and lecturer in mathematics at Milan; having heard of Tartalea's discoveries, and being about to publish a large work on mathematics, he desired to obtain them in order to add them to his treatise. And now began a series of intrigues worthy of mediæval Italians. Cardan first applied to Tartalea through a third person, offering services and friendship, but in vain. He next sought him by letter, but only obtained the roots of certain equations without the methods of finding them. Not to be beaten, Cardan approached Tartalea again, promising to use influence in his behalf with a certain nobleman resident in Milan, a patron of men of learning, whom he represented as being desirous of seeing him. Hope of patronage, or fear of giving offence in case of non-compliance, at length drew Tartalea to Milan, and, the nobleman being absent from the city, he was induced to remain three days at the house of Cardan. Here the latter at length obtained the rules without the demonstrations, not, however, without the most earnest entreaties and solemn oaths never to disclose the information. Cardan soon discovered for himself the demonstrations, and solved additional cases that had resisted the attacks of Tartalea, who in turn had recourse to various devices to obtain from Cardan these fresh discoveries. A violent quarrel ensued, which culminated in Cardan forgetting his oaths and promises, and publishing a treatise on cubic equations. Such were the circumstances attending this most important advance in the theory of equations. Posterity will readily forgive the offence, and laugh at the quarrel. The episode, however, is instructive as illustrating the curious, selfish view with which even scientific knowledge was regarded during the middle ages. Cardan effected the complete solution of cubic and biquadratic equations. He showed that the even roots of positive quantities are either positive or negative, the odd roots of negative quantities real and negative, and the even roots of negative quantities impossible. He knew that the number of positive roots is equal to the number of changes of sign; that impossible roots enter in pairs; how to form an equation having given roots; and how to transfer an equation so as to want a particular term. He frequently used letters to denote quantities. Mathematicians were then accustomed to put their rules into verse; Cardan followed the fashion. We need not be surprised to learn that the versification was awkward. The object of this custom was to assist the memory, an object much more effectually attained by the subsequent introduction of a literal notation, and of signs and symbols.

MANIFOLD SPACE.

The following remarks on Manifold Space, from the inaugural address of Mr. Spottiswoode, President of the British Association, will be of interest to many of our readers:

It may first be remarked that our whole experience of space is in three dimensions, viz., of that which has length, breadth and thickness; and if for certain purposes we restrict our ideas to two dimensions as in plane geometry, or to one dimension as in the division of a straight line, we do this only by consciously and of deliberate purpose setting aside, but not annihilating, the remaining one or two dimensions. Negation, as Hegel has justly remarked, implies that which is negatived, or, as he expresses it, affirms the opposite. It is by abstraction from previous experience, by a limitation of its results, and not by any independent process, that we arrive at the idea of space whose dimensions are less than three.

It is doubtless on this account that problems in plane geometry, which, although capable of solution on their own account, become much more intelligible, more easy of extension, if viewed in connection with solid space, and as special cases of corresponding problems in solid geometry. So eminently is this the case, that the very language of the more general method often leads us almost intuitively to conclusions which, from the more restricted point of view, require long and laborious proof. Such a change in the base of operations has, in fact, been successfully made in geometry of two dimensions, and although we have not the same experimental data for the further steps, yet neither the modes of reasoning, nor the validity of its conclusions, are in any way affected by applying an analogous mental process to geometry of three dimensions; and by regarding figures in space of three dimensions as sections of figures in space of four, in the same way that figures in plane are sometimes considered as sections of figures in solid space. The addition of a fourth dimension to space not only extends the actual properties of