10. Sum the series

(1) $\frac{1}{\sqrt{2}} - \frac{1}{3} + \frac{\sqrt{2}}{9} - \frac{2}{27} + \ldots$ to infinity.

(2) $3+6+11+20+27+\ldots$ to n terms.

SOLUTIONS TO ALGEBRA—HONORS.

1. $a^2 b^2 c^2$.

2. $(a+b)(a-b)^3 + (b+c)(b-c)^3 + (c+a)(c-a)^3 = 2(a+b+c)(a-b)(b-c)(c-a)$
$(a-b)(a+b)^2 + (b-c)(b+c)^2 + (c-a)(c+a)^2 = -(a-b)(b-c)(c-a)$

$\therefore$ *G.C.M.* $= (a-b)(b-c)(c-a)$.

L.C.M. $2(a+b+c)(a-b)(b-c)(c-a)$.

3. $x^{\frac{1}{2}} - 2\sqrt{-1} - x^{-\frac{1}{2}}$; $x^{\frac{1}{4}} - x^{-\frac{1}{4}}\sqrt{-1}$.

Assume $x^4 + 2ax^3 + bx^2 + 2cx + d = (x^2 + mx + n)^2$

Equating co-efficients of like powers of x $a = m$, $b = m^2 + 2n$, $c = mn$, $d = n^2$; $\therefore m = \frac{mn}{n} = \frac{m^2 + 2n - 2n}{m}$.

4. Book-work.

5. (1.) $x = 25 - 10\sqrt{6}$.

(2) We have

$$\frac{(x+l)^2 - a^2}{(x+m)^2 - a^2} = \frac{(x+m)^2 - b^2}{(x+l)^2 - b^2}.$$

Subtracting 1 from each side and dividing both sides by $(x+l)^2 - (x+m)^2$ we have

$$(x+l)^2 - b^2 + (x+m)^2 - a^2 = 0.$$

$$\therefore x = \pm\frac{\sqrt{\left\{a^2 + b^2 - (l-m)^2\right\}} - l - m}{2}$$

$(x+l)^2 - (x+m)^2 = 0$ gives $x = \frac{l+m}{2}$

(3.) $x = -\frac{3}{2}$.

(4.) $\left.\begin{array}{l} x = \frac{1}{4} \\ y = \frac{1}{3} \\ z = \frac{1}{2} \end{array}\right\}$

6. 525.

7. If a, $2b$, c be in *H. P.*, $2b = \frac{2ac}{a+c}$

$$\therefore b = \frac{ac}{a+c}$$

or, $ab + bc = ac$,
i.e., $a^2 = (a+c)(a-b)$
or, $c^2 = (c+a)(c-b)$.

8. $\frac{\lfloor 2n}{\lfloor n \lfloor n} = \frac{(1.3.5\ldots 2n-1)(2.4.6\ldots 2n)}{\lfloor n \quad \lfloor n}$

$= 2^n . \frac{(1.3.5\ldots 2n-1)\lfloor n}{\lfloor n \quad \lfloor n}$

$=$ &c.

9. $\frac{n(n+1)(n+2)(n+3)}{\lfloor 4}\left(2^3\right)^{-(n+4)}.2^4$

Put $n = \frac{1}{3}$, and the second part of question follows.

10. (1) $S = \frac{a}{1-r} = \frac{\sqrt{2}}{1 + \frac{\sqrt{2}}{3}} = \frac{3}{2 + 3\sqrt{2}}$

$S = (1^2 + 2) + (2^2 + 2) + (3^2 + 2) + \ldots + (n^2 + 2)$
$= \frac{n(n+1)(2n+1)}{6} + 2n$

PROBLEMS.

1. *AD*, *BE* are drawn at right angles to the opposite sides of the triangle *ABC*. Shew that the triangle *CED* is similar to the triangle *CBA*.

2. Produce a line so that twice the rectangle contained by the whole line and the part produced shall equal the square on half the given line.

3. *ABCDEF* is a hexagon inscribed in a circle such that *AB*=*BC*, *CD*=*DE*, and *EF*=*FA*. Shew that *AD*, *BE*, and *CF* pass through a point and are at right angles to *BF*, *FD*, and *DB* respectively.

IV. In any triangle inscribe a triangle similar to a given triangle.

V. Four straight lines form four triangles: the centres of the four circumscribed circles lie on a circle which also passes through the common point of the circumscribed circles.

VI. An *A.P.*, a *G.P.*, and an *H.P.* have each the same first and last terms, and the same number of terms (n), and the r^{th} terms are a_r, b_r, c_r; prove that

$$a_{r+1} : b_{r+1} = b_{n-r} : c_{n-r}.$$

VII. There are p suits of cards, each suit consisting of q cards numbered from 1 to q; prove that the number of sets of cards numbered from 1 to q which can be made from all the suits is p^q.