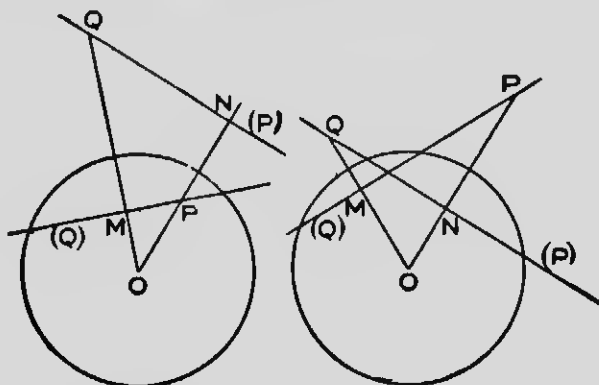


tended at the centre by any two points  $P$  and  $Q$  is equal to one of the angles which the polars of  $P$  and  $Q$  make with one another.

**PROP. 29.** If  $Q$  lie on the polar of  $P$ , then  $P$  must lie on the polar of  $Q$ .



Let  $(P)$  be polar of  $P$ , and let  $Q$  be any point on  $(P)$ .  
Then the polar of  $Q$  passes through  $P$ .

Join  $OQ$ , and draw  $PM$  perpendicular to  $OQ$ . Then the angles at  $M$  and  $N$  being right angles, a circle may be described to pass through the points  $P, M, Q, N$ .

Therefore  $OQ \cdot OM = OP \cdot ON = (\text{radius})^2$ .

Hence  $PM$  is the polar of  $Q$ , and  $P$  lies on the polar of  $Q$ .

**PROP. 30.** A chord of a circle is divided harmonically by any point on it and the polar of that point.

Let  $AB$  be any chord of the circle through  $P$ ; and let  $(P)$  be the polar of  $P$ , cutting  $AB$  in  $Q$ .

Then  $AB$  is divided harmonically in  $P$  and  $Q$ .