

rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

Produce a given line so that twice the rectangle contained by the whole line produced and the part produced may be equal to the square on the given line.

5. The angles in the same segment of a circle are equal to one another.

ABC is a triangle in the circle ABC ; AOD is drawn from A bisecting the arc BC in D and meeting the side BC in O ; prove that DB is a tangent to the circle through A, O, B .

6. If two straight lines cut one another within a circle, the rectangle contained by the segments of one of them shall be equal to the rectangle contained by the segments of the other.

Two circles with centres C and E touch each other internally at the point A ; from the centre C of the smaller circle CB is drawn at right angles to CE meeting the circumference of the larger circle in B ; and from E the centre of the larger circle ED is drawn parallel to CB meeting the circumference of the small circle in D ; prove that the straight line AB is equal to straight line AD .

7. To describe a circle about a given triangle.

O is the centre of the circle inscribed in the triangle ABC ; circles are described about the triangles BOC, COA, AOB , having as centres A', B', C' , respectively; prove that

$$AO.OA' = BO.OB' = CO.OC'.$$

8. Define similar rectilineal figures and reciprocal triangles.

ACB, ADB are two triangles upon the same base AB and between the same parallels; AD and BC meet in O ; shew that AOB, COD are similar triangles and COA, DOB are reciprocal triangles. Also, if EOF be drawn through O parallel to AB , shew that the quadrilaterals $CDOE, CDOF$ are equal.

9. In a right angled triangle, if a perpendicular be drawn from the right angle to the base, the triangles on each side of it are similar to the whole triangle, and to one another.

Construct a right angled triangle having given the hypotenuse and the difference of the squares on the two sides.

10. In equal circles, angles, whether at the centres or at the circumferences, have to one another the same ratio as the arcs on which they stand.

TRIGONOMETRY.

Examiner: CHARLES CARPMAEL, M.A.

1. Define the logarithm of a number, and explain what is meant by the "base" of a system of logarithms.

Shew that $\log \frac{a^n}{b^m} = n \log a - m \log b$.

Find $\log 175$, and $\log 6860$.

Of what numbers are 2, 0, $\bar{8}$, 0.25 the common logarithms?

2. Find the logarithm of the square root of $\frac{\sqrt{8} \cdot \sqrt[3]{577}}{49 \cdot \sqrt[3]{686}}$, and of $\frac{\sqrt[3]{-002}}{\sqrt[3]{-07}}$.

3. Define the terms sine, cosine, and tangent; and make a table of their variations in magnitude and algebraic sign from 0 to 180°.

Having given the tangent of an angle, find the sine and cosine.

4. Find the sine, cosine, and secant of 80° and 45°.

5. If ABC be a triangle, right-angled at C , shew how to find any of the quantities B, a, b , if A, c are given.

6. Prove that

$$\sin A \pm B = \sin A \cos B \pm \cos A \sin B.$$

$$\sin 3A = 3 \sin A - 4 \sin^3 A.$$

7. Prove the following formulæ:

$$\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \quad \sin 45^\circ + \theta \sin 45^\circ - \theta = \frac{1}{2} \cos 2\theta.$$

$$\sin^2 \theta - \sin^2 \phi = \sin \theta + \phi \sin \theta - \phi.$$

$$\frac{\sin \theta + \sin 3\theta}{\cos \theta + \cos 3\theta} = \tan 2\theta. \quad \tan 67^\circ 30' = 1 + \sqrt{2}.$$

8. In any triangle, prove

$$(i) \cos A = \frac{b^2 + c^2 - a^2}{2bc}. \quad (ii) \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

9. Solve completely the triangles:

(i) $a = 1263, b = 1859, c = 1468$.

(ii) $A = 67^\circ 59', a = 2045, b = 2000$.

10. Find the areas of the triangles in question 9.

11. The elevation of a tower is found to be 45°, and on retiring 60 yards it is 30°, find the height of the tower.

No.	Log.	No.	Log.	ANGLE.	L. SIN.
12680	.10140	20450	.31069	46°58'	9.86389
18590	.18322	30000	.47712	52°54'	9.90178
14680	.16678	57700	.76118	59° 7'	9.98360
14948	.17458	68600	.83682	65° 8'	9.95745
16124	.20747	70000	.84510	67°59'	9.96711
20000	.30103	78200	.89321		
		79561	.90070		

ALGEBRA AND TRIGONOMETRY—HONORS.

Examiner: CHARLES CARPMAEL, M.A.

1. Prove that

$$n^r - n(n-1)^r + \frac{n \cdot n-1}{2} (n-2)^r - \frac{n \cdot n-1 \cdot n-2}{3} (n-3)^r + \dots$$

$$\text{is } \begin{cases} n & \text{if } r=n, \text{ and} \\ 0 & \text{if } r \text{ be less than } n. \end{cases}$$

2. Enunciate and establish the principle of Indeterminate coefficients.

Employ it to find the sum to n terms of the series

$$1. 1^2 + 2(2^2 + 1^2) + 3(3^2 + 2^2 + 1^2) + \dots$$

3. Prove that a series is convergent if the ratio of each term to the preceding be less than some assignable quantity which is itself less than 1. Will this test apply if the ratio being always less than 1 approaches 1 as its limit?

Determine whether the series whose n^{th} term is $\sqrt{n^2 + 1} - n$ is convergent or divergent.

4. What are figurate numbers? Shew by induction that the figurate numbers of the r^{th} order are the successive coefficients of the binomial expansion of $(1-x)^{-r}$.

If f_r denote the r^{th} number of the q^{th} order, prove that

$$mf_{r+s} = f_r \cdot mf_s + f_{r-1} f_s + f_{r-2} f_{s+1} + \dots + mf_{r-1} f_s.$$

5. Shew that any convergent is nearer to a continued fraction than any other fraction having a smaller denominator than the convergent has.

If $\frac{p_n}{q_n}$ be any convergent to $\frac{1}{1+} \cdot \frac{2}{2+} \cdot \frac{3}{3+} \dots$, prove that $p_n + q_n = n+1$.

6. Find the number of divisors of a given composite number, and the sum of the cubes of these divisors.