You reply, perhaps, because there are only three dimensions in actual space. But in making hypotheses we need not limit ourselves to actualities; we can improve our methods of research, and gain clearer conceptions of the actual by passing outside and considering the possible.

For logical purposes there is no limit to the admissibility of hypotheses, provided we consider them purely as hypotheses, and do not teach that they are actual facts of the universe. It is, therefore, perfectly legitimate to inquire what our geometry would be if, instead of being confined to three dimensions, we introduced a fourth. Many curious conclusions follow. When we are confined to a plane a circle completely bounds a region within the plane, so that we cannot pass from the inside to the outside of the circle without intersecting it. Beings conscious only of two dimensions and moving only in two dimensions, and placed inside such a material circle, would find themselves completely imprisoned, with no possibility of getting outside. But give them a third dimension, with the power to move into it, and they simply step over the circle without breaking it. They do not have to even touch it. Living, as we do, in space of three dimensions, the four walls, pavement and ceiling of a dungeon, confine a person so completely that there is no possibility of escaping without making an opening through the bounding surface. But give us a fourth dimension, with the faculty of moving into it, and we pass completely outside of our three dimensional universe, by a single step, and get outside the dungeon as easily as a man steps over a line drawn on the ground. Were motion in the fourth dimension possible, an object moving in that dimension by the smallest amount would be completely outside of what we recognize as the universe, and would, therefore, become invisible. It could then be turned around in such a way that on being brought back it would be obverted, or appear as in a looking glass. A man capable of such a motion would come back into our sight similarly obverted, his left side would now be his right, without any change having taken place in the relative positions of the particles of his body. The somerset he would have turned would have completely obverted every atom and molecule of his body without introducing any disturbance into its operations.

This possibility of obversion brings in a curious question concerning the rigor of one of the fundamental propositions in elementary geometry. Euclid proves by superposition that the two triangles in a plane having two angles and the included side equal are equal to each other. In the demonstration it is assumed that the triangles can be made congruent by simply placing one upon the other without taking it out of the plane. From this the conclusion is drawn that the same conclusion holds true if one of the triangles be obverted. But in this case they cannot be brought into congruence without taking one of them out of the plane and turning it over. The third dimension is thus assumed in geometry involving only two dimensions.

Now consider the analogous case in space. Two pyramids upon congruent bases may be proved equal by bringing them into congruence with each other. But suppose that they differ only in that one is the obverse of the other, so that they could be brought into congruence only by looking at one of them in a mirror and then placing the other into congruence with the image of the first as seen in the mirror. Would we detract from the rigor of the demonstration by assuming the possibility of such an obversion without changing the volume of the pyramid? With a fourth dimension we should have no detraction from rigor. We would simply obvert the