ments with a slight pause between each movement. Each little movement of the gate would destroy instantaneously a small part, ΔV, of the velocity, V_0, and, since this part of the velocity is destroyed instantaneously, the rise of pressure, according to Joukovsky, would be $h = a\Delta V/g$. When the first instantaneous movement has taken place, let a pause of time, t, elapse before the next movement. Then $\Delta V = V_0 - V_t$, and $h_t = a(V_0 - V_t)/g$.

By these equations and Equation (4) a numerical example may now be solved, and at the same time there will be explained other interesting phenomena caused by the closing of the gates and the pressure waves produced thereby. The analytical work may then be more readily understood.

Let $L = 820$ ft.;
$V_0 = 11.75$ ft. per sec.;
$H_0 = 165$ ft.;
$T = 2.1$ sec. (For convenience, T has been chosen an even multiple of $2L/a$).
$a = 4{,}680$ ft. per sec. (This value of a is chosen simply because this example was worked out by the writer for a pipe in a tunnel and concreted in. The expansion of the pipe walls, therefore, was neglected. For any condition, a may be obtained by Equation (2).

Since $V_0 = B_0(H_0)^{1/2}$, then $B_0 = 0.91476$.

Assume that the gates are closed in 24 successive instantaneous movements. The time elapsing between each movement would then be 0.0875 sec. After the first of these movements had taken place the gates would have been closed one-twenty-fourth of their opening, and the number representing the gate opening would have been reduced by one-twenty-fourth of its value, that is, 0.91476/24 = 0.038115. At each of the successive movements the value of B is reduced by the same amount, as the gate motion is assumed to be uniform. It will be necessary, however, to use more than the first three significant figures, and the work may be done on the slide-rule. In this example the recovery of the friction head in the penstock will be neglected.

From the foregoing may now be written the first three columns of Table 1, and the first line of Columns 4 and 5. The table may then be completed as follows: Assume a trial reduction in velocity, caused by the initial instantaneous movement of the gate, and set the figure down in Column 5 under the value of V_0 and subtract it from V_0, placing the difference immediately underneath. This trial figure is ΔV, and is assumed to be destroyed instantaneously by the first movement of the gate. A pressure wave, Δh, of magnitude, $a\Delta v/g = 4{,}680(\Delta V)/32.2 = 145\Delta V$, is therefore started up the pipe. The product of 145 ΔV is set down in Column 6 opposite ΔV. In Column 7 is recorded the algebraic sum of the value of Δh. Having obtained the figure in Column 7, it is added to the net head, $H_0 = 165$, and the sum is set down in the next line lower in Column 4. The result must now be checked, to see that $B(H)^{1/2} = V$, where B is 0.8766 and H and V have the values recorded in their respective columns opposite B. If the relation is not satisfied, a new trial value of ΔV must be chosen, and the operations repeated until a check is obtained. After trial, the initial value of ΔV was found to be 0.085. Proceeding in this way, the rise of pressure at the end of 0.35 sec. is found to be 56.80 ft. This rise has taken place in four successive jumps.

At this point it becomes necessary to trace the course of the pressure wave started up the penstock by the initial movement of the gate. This wave has a velocity of 4,680 ft. per sec., and travels at that rate toward the forebay or origin of the penstock; after arriving at the origin it is reflected and returns to the gate at the same velocity. The distance from gate to forebay and return is 1,640 ft., so that the pressure wave takes 0.35 sec. to cover this distance. On its arrival at the gate it is reflected immediately as a wave of sub-normal pressure, and commences its journey again from gate to forebay and back. At the instant the wave becomes sub-normal, however, the gate is given one of its instantaneous closing movements, causing a further reduction in the velocity of the water flowing in the penstock and the consequent rise of pressure incident thereto. Thus, at this instant, two factors have to be taken into consideration: the rise of pressure caused by the fifth little instantaneous movement of the gate and the fall in pressure caused by the change from super-normal to sub-normal of the pressure wave produced by the first or initial movement of the gate which occurred 0.35 sec. before.

By trial-and-error the velocity that has been destroyed by the fifth movement of the gate is found to be 0.258 ft. per sec., and the result is checked as follows. Multiplying 0.258 by 145, the magnitude of the resulting pressure wave is 37.4 ft., and this added to 56.80 would make the total excess pressure existing equal to 94.20 ft., were it not for the fact that the initial wave has returned to the gate and become sub-normal. The amount of the initial wave, as shown by the second line of Table 1, is 12.32, and since it not only falls to

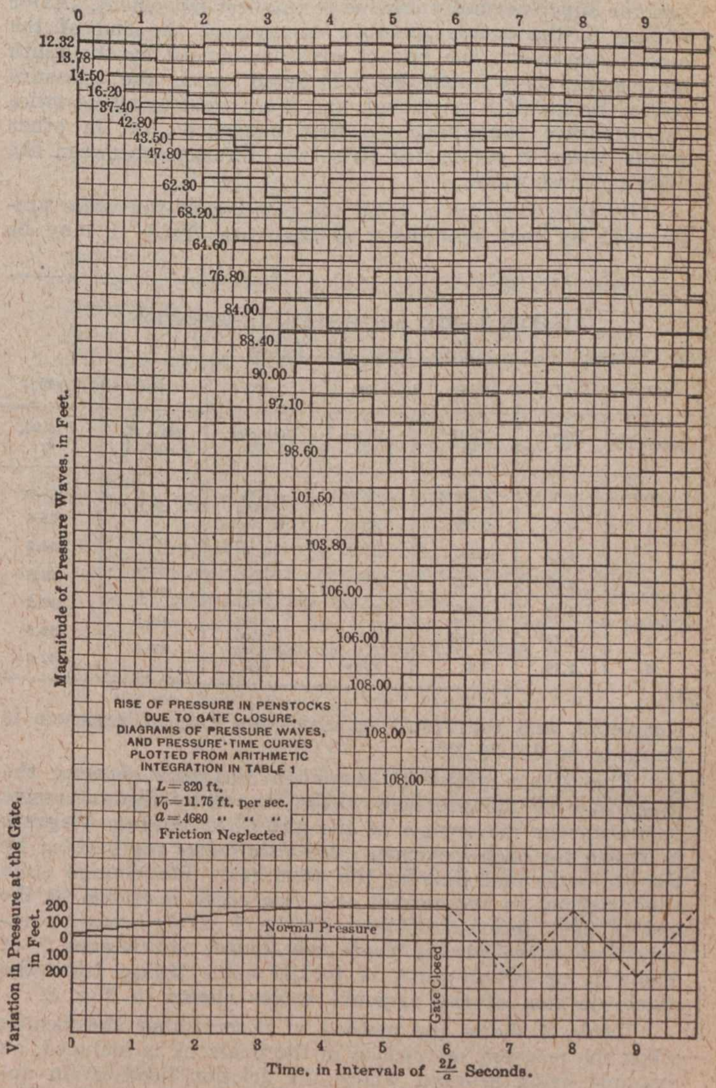

Fig. 2—Diagrams of Pressure Waves and Pressure-Time Curves Plotted from Table 1

zero but passes below zero to a sub-normal pressure of equal amount, there must be subtracted twice 12.32 from 94.20, making the net excess pressure existing at that instant 69.56 ft. Adding this to the initial net head $H_0 = 165$, the value of V is checked as before by the formula, $V = B(H)^{1/2}$.

Proceeding then as before, and remembering that the pressure rise caused by the sixth movement of the gate is reduced by the fall to sub-normal of the wave produced by the second movement of the gate, and the pressure rise of the seventh movement is reduced by the wave produced by the third movement, and so on, there may be calculated the successive increments of pressure.

For convenience, the time required by the pressure waves to travel from the gate to the origin of the pipe and return to the gate again will be called one interval. The inter-