From (1) $b+c > \pm\ a$
(2) $a > \pm(b-c)$;
or, $b+c>a$; $b+c+a>0$; $a>b-c$, and $a>c-b$; or, $(a+b+c)>0$; $b+c-a>0$; $a-b+c>0$, and $a-c+b>0$.

91. If $(-1)^{\frac{1}{3}}+(-1)^{-\frac{1}{3}}=y$, show that its value is given by the equation $y^3-3y+2=0$, and solve this equation.

Let $(-1)^{\frac{1}{3}}=x$ and $(-1)^{-\frac{1}{3}}=x^{-1}$ then $x+x^{-1}=y$, and cubing this equation $x^3+x^{-3}+3(x+x^{-1}) = x^3+x^{-3}+3y=y^3$; but $x^3=1$ and $x^{-3}=-1$.

$\therefore 2+3y=y^3. \qquad y^3-3y+2=0;$

the values are, $y=1$, 1, and -2.

EDGAR FRISBY, M.A.,
Naval Observ., Washington.

---

124. What is the length of an edge of the largest cube that can be cut from a sphere 40 inches in diameter?

The diameter of the circle is a diagonal of the cube; therefore this diagonal is the hypothenuse of a right-angled triangle, one side of which is an edge of the cube and the other side a diagonal of a face of the cube. This last side is an edge of the cube multiplied by $\sqrt{2}$; therefore the first diagonal is an edge of the cube multiplied by $\sqrt{3}$; then $40 \div \sqrt{3}$ gives the edge desired, which is 23·08.

125. Any number is divisible by 11, if, counting from the units place, the sum of the digits in the odd places is equal to the sum of the digits in the even places, or if the difference between these sums is divisible by 11. Give mathematical proof of this.

Let p, q, r, s, etc., be the digits composing the given number, and suppose that p occupies the sixth place from the decimal point, q the fifth, and so on: then the number will be $(10)^6 p+(10)^5 q \dots\dots (10)u+v$; or in a different form: $(11\text{-}1)^6 p+(11\text{-}1)^5 q+(11\text{-}1)^4 r+\dots\dots(11\text{-}1)u+v$. Expanding in each case, it is seen that 11 is a factor of every term except the right hand ones of each expansion, and these last terms are of the form $+p$; $-q$; $+r$; $-s$; $+t$; $-u+v$. Now if the sum of these terms is 0, or is divisible by 11, the sum of the expansions, and consequently the whole number is divisible by 11.

126. A circular grass plot, whose area is one-quarter of an acre, has erected at its centre a pole 12 feet high, and of the uniform diameter of one foot. Attached to the top of this pole is one end of a cord, the length of which is just sufficient to allow the other end to touch the edge of the plot. The cord is then wound spirally on the post so as to make one complete revolution in every foot of its descent. When it has been thus wound from the top to the bottom of the post, what is the area of the circle, in square yards, of which the unwound part of the cord is radius.

The length of the string is obtained by finding the hypothenuse of a right-angled triangle whose perpendicular is 12 feet and base the radius of the circle. Taking π as 3·1416, we find R = 58·86 feet, and the string is 60·67 feet long. Now the length of string required to make one spiral on the post is the length of the hypothenuse of a right-angled triangle whose perpendicular is one foot, and base the circumference of the post. For one spiral there is wanted 3·29 feet, and for 12 spirals 39·48 feet. Now 60·67 − 39·48 = 21·19, which is the radius of the second circle. From this the area is found to be 1410·61 feet or 156·73 yards.

127. If x be any odd number greater than unity, shew that (x^5-x) is divisible by 24; also that $(x^2+3)(x^2+7)$ is divisible by 32.

$(x^3-x)=x(x-1)(x+1)(x+2)$, that is, arranging them in ascending order $(x-1)\,x\,(x+1)(x+2)$ or 4 consecutive integers. Now if x be *odd*, $(x-1)$ is divisible by 2, $(x+1)$ is divisible by 2, and one of their quotients is again divisible by 2: therefore the product of the divisors thus far is 8. If x be *even* the same reasoning applies to x and $(x+2)$, but some one of any three consecutive integers is divisible by 3, therefore the whole number is divisible by 24.

[This also follows from the general proposition: "that the product of n consecutive integers is divisible by $\underline{|n}$."]