

Hence, by the first two of equations (8),

$$B + B'\sqrt{z} = -\frac{13}{100}\left(3 + \frac{7\sqrt{5}}{5}\right).$$

Therefore, for the equation (32), the values of u_1^5 , u_4^5 , u_2^5 and u_3^5 are

$$\begin{aligned} u_1^5 &= -\frac{13}{100}\left(3 + \frac{7\sqrt{5}}{5}\right) - \sqrt{\left\{\frac{13^3}{100^3}\left(3 + \frac{7\sqrt{5}}{5}\right)^2 + \frac{\sqrt{5}}{625^2}\right\}}, \\ u_4^5 &= -\frac{13}{100}\left(3 + \frac{7\sqrt{5}}{5}\right) + \sqrt{\left\{\frac{13^3}{100^3}\left(3 + \frac{7\sqrt{5}}{5}\right)^2 + \frac{\sqrt{5}}{625^2}\right\}}, \\ u_2^5 &= -\frac{13}{100}\left(3 - \frac{7\sqrt{5}}{5}\right) - \sqrt{\left\{\frac{13^3}{100^3}\left(3 - \frac{7\sqrt{5}}{5}\right)^2 - \frac{\sqrt{5}}{625^2}\right\}}, \\ u_3^5 &= -\frac{13}{100}\left(3 - \frac{7\sqrt{5}}{5}\right) + \sqrt{\left\{\frac{13^3}{100^3}\left(3 - \frac{7\sqrt{5}}{5}\right)^2 - \frac{\sqrt{5}}{625^2}\right\}}. \end{aligned}$$

p_2 not assumed to be zero.

§17. Let us now consider the more general case in which p_2 is not assumed to be zero. The method that has been illustrated above is still applicable, though the labor of operation, in dealing with particular instances, is increased.

Putting, as before, y for $\alpha^2 z$, and t for $\frac{c}{a}$, we form equations corresponding to (11) and (16); from these we obtain the values of y and t ; then we find B and $B'\sqrt{z}$ from equations corresponding to the first two of the group (8); or, B can be more readily found from the second of equations (6), and, on the principles of §9, when $B + B'\sqrt{z}$ is known, $u_1 u_4$ or $g^2 - y$ being also known, the root of the given quintic is known.

§18. The values of B and $B'\sqrt{z}$ which correspond, when g or $-\frac{p_2}{10}$ is not zero, to those given in (8) for the case in which g is zero, are obtained from the equations (2) and (3) by keeping in view that, according to the first of equations (4), B is the rational part and B' the coefficient of $\sqrt{z}$ in the expansion of u_1^5 .

Put

$$\begin{aligned} A' &= ch(\theta^2 - \phi^2 z) \\ P &= 2k(k^2 - t^2 y) - (g^2 - y)(gk - ty) \\ Q &= 2t(k^2 - t^2 y) - (g^2 - y)(k - gt) \end{aligned} \quad (33)$$

Then

$$(g^2 - y)^2 B = (g^2 + y)P + 2gyQ + 2azA'\{t(g^2 + y) + 2gk\}$$

and

$$(g^2 - y)^2 B' = a\{2gP + (g^2 + y)Q\} + 2A'\{k(g^2 + y) + 2gt y\} \quad (34)$$