8. Two equal circles touch a straight line at A and B, and do not intersect; and on each of them at equal intervals are situate $2n+1$ points, A and B being such points. The only lines that contain more than two of the points are those that are parallel to AB. Find the number of triangles that can be formed by joining these points—both circles being utilized for each triangle.

The number of pairs of points on the circle A is the number of combinations of $2n+1$ things 2 together, that is, $n(2n+1)$. If now we suppose that no line contains more than two points, then each of the $n(2n+1)$ pairs of points in A will form a triangle with each of the $2n+1$ points in B; there will $\therefore$ be $n(2n+1)(2n+1)$ triangles, each of which has *one* angular point on the circle B. Similarly then will be the same number of triangles having one angular point on A; that is, there would be $2n(2n+1)(2n+1)$ triangles altogether. But each one of the n lines parallel to AB prevents the formation of four triangles, $\therefore$ our first result must be diminished by $4n$, which gives

$$2n(4n^2+4n-1).$$

9. Show how to determine the greatest term in the expansion of $(a+x)^n$.

10. (1) The coefficient of x^r in the expansion of

$$(1-x)^{-\frac{3}{2}} \text{ is } \frac{\underline{|2r+1}}{\underline{|r}\ \underline{|r}} \cdot \frac{1}{2^{2r}}.$$

The expansion of $(1-x)^{-\frac{3}{2}}$ is

$$1+3\left(\frac{x}{2}\right)+\frac{3.5}{1.2}\left(\frac{x}{2}\right)^2+\frac{3.5.7}{1.2.3}\left(\frac{x}{2}\right)^3+\ldots$$

$\therefore$ coefficient of x^r is

$$\frac{3.5.7.9.\ldots(2r+1)}{2^r\ \underline{|r}}, \ (a).$$

Now since $2.4.6.\ldots 2r$

$$=2^r(1.2.3\ldots r)=2^r\ \underline{|r}$$

if we multiply the num'r of (a) by $2.4.6.\ldots r$ and its denominator by $2^r\ \underline{|r}$, the value of (a) will not be altered and it becomes

$$\frac{1.2.3.4.5.6.\ldots 2r+1)}{2^{2r}\ \underline{|r}\ \underline{|r}}$$

(2) If a_r be the coefficient of x^r in the expansion of $(1+x)^n$; then, n being a positive integer,

$$\frac{a_1}{a_0}+\frac{2a_2}{a_1}+\frac{3a_3}{a_2}+\ldots+\frac{na_n}{a_{n-1}}$$

$$=\frac{n}{2}(n+1).$$

We have $a_0=1$, $a_1=n$,

$$a_2=\frac{n(n-1)}{1.2}, a_3=\frac{n(n-1)(n-2)}{1.2.3}, \ldots$$

$$\therefore \frac{a_1}{a_0}=n,$$

$$\frac{2a_2}{a_1}=n-1,$$

$$\frac{3a_3}{a_2}=n-2,$$

&c. = &c.

$$\frac{na_n}{a_{n-1}}=1,$$

$\therefore$ the sum required is the sum of the first n natural numbers, or

$$\frac{n}{2}(n+1).$$

MECHANICS.—*1st C.*

1. Define a couple and show that the forces composing one do not admit of a single resultant.

State the various transformations that may be made on a couple without alteration of effect. Establish the truth of one of them.

The sides of a quadrilateral are acted on by forces perpendicular to them, and proportional to them in magnitude, the forces being turned inwards. Show that if the points of application divide the sides in a constant ratio they reduce to a couple.

This result will readily appear if the forces be resolved at right angles and along one of the sides. The forces may produce equilibrium.

2. Find the centre of gravity (1) of a triangular area; (2) of three uniform rods forming a triangle.