TRIGONOMETRY – Honors.

PROFESSOR A. R. BAIN (*Victoria*), EXAMINER.

1. If the angle at the centre of a circle which is subtended by the side of a regular octagon be taken as the unit of measure, by what number is that angle represented whose circular measure is $\frac{22}{7}$ or π?

2. If the minute-hand of a clock is 2 feet 11 inches in length, and if $\pi = \frac{22}{7}$, how far will the point of the hand move in 12 minutes?

3. Find θ in each of the following equations:

$$(\alpha)\ 2\tan^2\theta + \sec^2\theta = 2.$$

$$(\beta)\ \sin\theta\cos\theta + \sin^2\theta = \frac{1+\sqrt{3}}{4}.$$

4. The $\cot\varphi = 1+\sqrt{2}$; find all the other functions of φ, supposing φ to be in the third quadrant.

5. Prove the following quantities:

$$(a)\ \frac{\cos(\alpha+\beta+\gamma)}{\cos\alpha\cos\beta\cos\gamma} = 1 - \tan\alpha\tan\beta - \tan\beta\tan\gamma - \tan\alpha\tan\gamma.$$

$$(b)\ \frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \tan 3x.$$

$$(c)\ 1 + \tan x \tan\frac{1}{2}x = \sec x.$$

6. Show that $\tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{3} = 45°$.

7. Prove that in any triangle $\cos A = \frac{b^2+c^2-a^2}{2bc}$, and then find the formula which gives the value of $\tan\frac{1}{2}A$ in terms of the sides.

8. In a given triangle $A = 30°$, $B = 60°$ and $C = 90°$, and the perpendicular from C upon $c = q$. Show that the area of the triangle $= \frac{2q^2}{\sqrt{3}}$.

9. The angle of elevation of a tower is observed. At a point a feet nearer, the elevation is the complement of the former; b feet nearer still, it is double the first elevation. Show that the height of the tower is $\sqrt{(a+b)^2 - \frac{a^2}{4}}$.

10. If the angles A, B, C of the triangle ABC be as 2, 3, 4 respectively, then $2\cos\frac{A}{2} = \frac{a+c}{b}$.