MULTIPLICATION.

1. Wherever practicable multiplication should be performed in *one line* ; by reducing the quantities to be operated upon to the form of binomial factors the multiplication can be readily effected by referring it to the results obtained by the multiplication of well known simple forms.

Fundamental Forms.

i. $(a \pm b)^2 = a^2 + b^2 \pm 2ab.$

ii. $(a+b)(a-b) = a^2 - b^2.$

iii. $(x+a)(x+b)(x+c) = x^3 + x^2 \,.\, \overline{a+b+c}$
$\qquad + x \,.\, \overline{ab+bc+ca} + abc.$

iv. $(mx+a)(nx+b) = mnx^2 + \overline{mb+na} \,.\, x + ab.$

Ex. 1. $(a+b+c)^2 = a^2 + (b+c)^2 + 2a(b+c)$
$\qquad = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.$

2. $(x + \frac{1+\sqrt{3}}{2})(x + \frac{1-\sqrt{3}}{2}) = (x + \frac{1}{2} + \frac{1}{2}\sqrt{3})$
$\qquad (x + \frac{1}{2} - \frac{1}{2}\sqrt{3}) = (x+\frac{1}{2})^2 - \frac{3}{4} = x^2 + x - \frac{1}{2}.$

3. $(x^2 + ax - b)(x^2 - ax + b) = x^4 + (\overline{ax-b} + \overline{-ax+b})x^2$
$\qquad + (ax-b)(-ax+b) = x^4 - a^2x^2 + b^2.$

4. $(a+b-c+d)(a-b+c-d) = (\overline{a+b} - \overline{c-d})(\overline{a-b} + \overline{c-d})$
$\qquad = (a+b)(a-b) + (a+b)(c-d) - (a-b)(c-d) - (c-d)^2$
$\qquad = a^2 - b^2 - c^2 - d^2 + 2bc - 2bd + 2cd.$

5. $3(p+q)(q+r)(r+p) = 3(p+q)(qr + qp + r^2 + rp)$
$\qquad = 3(p^2q + pq^2 + p^2r + pr^2 + q^2r + qr^2 + 2pqr).$