ALGEBRA.

1. Find the factors of $x^3 + y^3 + z^3$ when $x + y + z = 0$.

Find the binomial expression which equated to zero will make

$$x^3 - (2a + b)x^2 + (2ab + a^2)x - a^2b$$

vanish.

2. Without simplifying

$(a + b + c)(ab + bc + ca) - (a + b)(b + c)(c + a)$,

show that it is equal to abc.

3. Find the H.C.F. of

$$x^3 - 2x^2y + 4xy^2 - 8y^3$$

and $x^3 + 2x^2y + 4xy^2 + 8y^3$;

and the L.C.M. of

$$(a + b)\left\{(a + b)^2 - c^2\right\}$$

and $4b^2c^2 - (a^2 - b^2 - c^2)^2$.

4. Simplify

(1) $$\frac{(x+a)(x+b)}{(b-c)(c-a)} + \frac{(x+b)(x+c)}{(c-a)(a-b)} + \frac{(x+c)(x+a)}{(a-b)(b-c)}.$$

(2) $$s(s-a)(s-b) + s(s-b)(s-c) + s(s-c)(s-a) - (s-a)(s-b)(s-c)$$

where $2s = a + b + c$.

5. Solve the equations:

(1) $$\frac{1}{x-1} + \frac{1}{x-2} = \frac{2}{x}.$$

(2) $$\frac{x}{a+b-c} + \frac{x}{a+b+c} = \frac{a+b}{a^2+b^2-c^2+2ab}.$$

(3) $$\frac{x}{(a-b)(b-c)} + \frac{x}{(b-c)(c-a)} + \frac{x}{(c-a)(a-b)} = \frac{1}{(a-b)(b-c)(c-a)}.$$

6. A man buys a shares of a certain stock for b dollars, and sells c shares of the stock at an advance of p per cent. At what price should he sell the remainder so as to gain $3p$ per cent. on the whole?

7. Extract the square root of

$$\frac{x}{y} + \frac{y}{x} + x + y + 2 + \frac{x}{\sqrt{y}} + \frac{y}{\sqrt{x}} + \sqrt{x} + \sqrt{y} + 2\sqrt{xy},$$

and the cube root of $a^6 + x^6$ to three terms.

8 (1) If $a^4 + \frac{1}{a^4} = a^2 + \frac{2}{a^2} + 2$, determine the value of $a^3 + \frac{1}{a^3}$.

(2) If

$$ax^2 + by^2 + cz^2 + 2a'yz + 2b'zx + 2c'xy = 0,$$

and $(ax + c'y + b'z)^2 = Ay^2 + Byz + Cz^2$,

determine A, B, C.

9. Solve the equations:

(1) $$\begin{cases} \frac{1}{x} + \frac{2}{y} = 8, \\ 6xy = 1. \end{cases}$$

(2) $x^3 + xy^2 = 16 = y^3 + x^2y$.

(3) Find values for x and y which will make $x^2y + y^2 + x$ and $xy^2 - x^2 + y$ simultaneously vanish.

10. There are three numbers; their sum is equal to their product; the sum of the first and third is half the second; and the product of the first and second, less the first, is equal to the third. Find the numbers.

EUCLID.

1. State the different conditions of equality of two triangles, as given in the first book of Euclid.

2. If two triangles have two sides in one equal to two sides in the other, each to each, and an angle, opposite an equal side, equal in each, are the triangles necessarily equal? Explain.

3. Any two sides of a triangle are together greater than the third side.

4. If a point be taken within a triangle and lines be drawn from it to the extremites of the base, the sum of these lines is less than the sum of the two sides of the triangle.

5. The three angles of a triangle are together equal to two right angles.

6. Three unlimited straight lines intersect one another not in a common point. What is the sum of all the angles formed?

7. If a line be divided into two equal and also into two unequal parts, the square upon the greater unequal part is equal to the square upon the less unequal part, together with four times the rectangle contained by the half line and the line between the points of section.

8. ABC is a triangle, and CD bisects the base AB in D. Show that $AC^2 + CB^2 = 2AD^2 + 2DC^2$.

9. Show how to construct a square equal to a given triangle.