Solutions by D. F. H. WILKINS, B.A., Mathl. Master, High School, Chatham.

163. If

$$\left.\begin{array}{l} cx+by-lxy=0\\ ay+cz-myz=0\\ bz+ax-nzx=0\end{array}\right\},$$

then $l^2acx+m^2bay+n^2cbz=0$, where l, m, n are the roots of the equation $f^3+g=0$.

Given $$\frac{c}{y}+\frac{b}{x}=l,$$

$$\frac{a}{z}+\frac{c}{y}=m,$$

$$\frac{b}{x}+\frac{a}{z}=n,$$

adding $2\left(\frac{b}{x}+\frac{c}{y}+\frac{a}{z}\right)=l+m+n$,

$\therefore\ \frac{b}{x}=\frac{l-m+n}{2}$, &c.

Now $l^2cax+m^2aby+n^2bcz$

$$=abc\left(l^2\frac{x}{b}+m^2\frac{y}{c}+m^2\frac{z}{a}\right)$$

$$=2abc\left(\frac{l}{l-m+n}+\dots\right),$$

$$=2abc\left(\frac{2(l^3n+\dots-l^4-\dots}{(l-m+n)(l+m-n)(-l+m+n)}\right).$$

$=0$ if $2(l^3n+\dots)-l^4-\dots=0$.

Now $2(l^2m^2+\dots\dots)-l^4-\dots\dots$

$$=(l+m+n)(l+m-n)$$
$$(l-m+n)(-l+m+n)=0,$$

$\therefore\ l+m+n=0$.

For, l, m, n being roots of the $=nf^3+g=0$

$l+m+n=0$, (1) $lm+mn+nl+0$. (2)

and $l^2m^2+\dots=\left\{(lm+\dots)^2-2lmn(l+\dots)\right\}=0$

$\therefore\ -l^4-m^4-n^4=0$.

and $2(l^3n+m^3n+n^2l)=0$.

For it equals $2lmn\left(\frac{l^2}{m}+\frac{m^2}{n}+\frac{n^2}{l}\right)$

Finding values of l and m from (1) and (2) in terms of n, we have

$$l=-\frac{n}{2}(1\pm\sqrt{-3}),\quad m=-\frac{n}{2}(1\pm\sqrt{-3})$$

$$\therefore\ \frac{l^2}{m}=n,\quad \frac{m^2}{n}=-\frac{n}{2}(1\pm\sqrt{-3}),$$

$$\frac{n^2}{l}=-\frac{n}{2}(1\mp\sqrt{-3})$$

and their sum $=0$.

$\therefore\ l^2cax+m^2aby+n^2bcz=0$.

169. Prove that

$$50\left\{x-y)^7+(y-z^7+(z-x)^7\right\}^2$$
$$=49\left\{(x-y)^4+\dots+\right\}\left\{(x-y)^6+\dots\right\}^2.$$

If $a+b+c=0$, there are the following easily proved relations :—

$a^2+b^2+c^2+2(bc+ca+ab=0$ (1).

$a^3+b^3+c^3-3abc=0$ (2).

$a^4+b^4+c^4+(bc+\dots)(a^2+\dots)=0$ (3).

$a^5+b^5+c^5+(bc+\dots)(a^3+\dots)-abc(a^2+\dots)=0$ (4).

$a^7+b^7+c^7+(bc+\dots)(a^5+\dots)-abc(a^4+\dots)=0$ (5).

From (1) $bc+ca+ab=-\frac{a^2+b^2+c^2}{2}$.

" (2) $abc=\frac{a^3+b^3+c^3}{3}$.

" (3) $a^4+b^4+c^4=\frac{(a^2+b^2+c^2)^2}{2}$ (6)

$\therefore$ (4) becomes

$$a^5+b^5+c^5=\frac{a^2+b^2+c^2}{2}(a^3+\dots)$$
$$+\frac{a^2+b^2+c^2}{3}(a^3+\dots)$$

$$\therefore\ \frac{a^5+b^5+c^5}{5}.$$
$$\frac{a^2+b^2+c^2}{2}\cdot\frac{a^3+b^3+c^3}{3}. \quad (7)$$

From (5) $a^7+b+{}^7c^7$

$$=\frac{a^2+b^2+c^2}{2}(a^5+b^5+c^5)$$

$$+\frac{a^3+b^3+c^3}{3}\cdot\frac{a^2+b^2+c^2}{2}\cdot(a^2+b^2+c^2).$$

$$=\frac{a^2+b^2+c^2}{2}\cdot(a^5+\dots)$$
$$+\frac{a^5+b^5+c^5}{5}\cdot(a^2+\dots)\text{. by (7)}$$