

then $a + b = 1 + 3 + 5 + 7 + \&c.$ to $2n$ terms $= 4n^2$ and $a - b = -2 - 2 \&c.$ to n terms $= -2n$

$$\therefore (a - b)^2 = 4n^2.$$

Next let b contain $n - 1$ terms, then $a + b = 1 + 3 + 5 + \&c.$ to $2n - 1$ terms $= (2n - 1)^2$, and $a - b = 1 + (2 + 2 + \dots$ to $n - 1$ terms $= 2n - 1$

$$\therefore (a - b)^2 = (2n - 1)^2$$

14. The roots of $ax^2 - bx = a^2x - ab$ are a and $\frac{b}{a}$

$$15. (1) x = \frac{-a - b \pm \sqrt{(a - b)^2 + 4nab}}{2}$$

$$(2) 6x - \sqrt{x} - 1 = (3\sqrt{x} + 1)(2\sqrt{x} - 1) \\ \therefore \sqrt{x} = \frac{1}{2} \text{ or } -\frac{1}{2}.$$

(3) Square both sides, then

$$2x - 1 + 2\sqrt{x^2} - x = x + 1$$

$$\therefore x - 2 = -2\sqrt{x^2} - x$$

square both sides

$$\therefore x^2 - 4x + 4 = 4x^2 - 4x$$

$$\therefore x = \pm \frac{2}{\sqrt{3}}$$

$$(4) x + y + z = \frac{a^2}{x} = \frac{b^2}{y} = \frac{c^2}{z} \\ = \frac{a^2 + b^2 + c^2}{x + y + z} \\ = \sqrt{a^2 + b^2 + c^2}$$

$$\therefore x = \frac{a^2}{\sqrt{a^2 + b^2 + c^2}}$$

Toronto University, Pass Algebra, First Year, 1860.

1. What extensions of the arithmetical definitions of the signs $-$, $\times$, are made in Algebra?

Prove $3 \times 2 = 2 \times 3$ and explain wherein this differs from the algebraic identity $ab = ba$.

2. If a number be multiplied by 4 and the same number reversed be multiplied by 5, the sum of the products is exactly divisible by 9.

Prove this and infer the general proposition of which it is a particular case.

3. Perform the following operations:

(1). Simplify

$$(a+b)(b+c) - (a+1)(c+1) - (a+c)(b-1);$$

(2). Multiply

$$\frac{x^2y^2}{9} - xy + 9 \text{ by } \frac{xy}{3} + 3;$$

$$(3). \text{ Divide } (ax + by)^2 + (cx + dy)^2 + (ay - bx)^2 + (cy - dx)^2 \text{ by } x^2 + y^2;$$

(4). Extract the square root of

$$a^2(x^2 + 4) - 2a(x + 2) + 4a^2x + 1;$$

(5). Find the highest common divisor of

$$a^2 + b^2 - c^2 + 2ab, \text{ and } a^2 - b^2 - c^2 + 2bc.$$

4. Describe Horner's method of synthetic division, and shew how to use it when the leading coefficient of the divisor is different from unity. Apply it in the following cases.

(1). Divide $4x^4 + 5x^2 + 1$ by $x^3 + 2x - 1$, obtaining the exact remainder, and also four terms of the remainder expressed in descending powers of x .

(2). Expand $\frac{1}{1 - x + x^2}$ in ascending powers of x .

(3). Find the remainder after the division of $8x^3 - 6x + 5$ by $2x + 3$.

5. Prove that the value of a fraction is unaltered by multiplying or dividing both numerator and denominator by the same quantity, and examine whether the value is increased or diminished by adding or subtracting the same quantity to or from both numerator and denominator.

6. Perform the following operations:

(1). Multiply

$$\frac{a}{a+b} + \frac{b}{a-b} \text{ by } \frac{a}{a-b} - \frac{b}{a+b};$$

(2). Divide

$$\frac{a}{a+c} - \frac{b}{b+c} \text{ by } \frac{c}{b+c} - \frac{c}{a+c};$$

(3). Reduce to a single fraction in its lowest terms:

$$\frac{3(x-2)}{(x-1)(x-3)} - \frac{1}{x-1} - \frac{1}{x-2} - \frac{1}{x-3};$$

$$(4). \text{ Prove } \frac{(x+1)(y+1)}{(x-1)(y-1)} =$$