

PROPOSITION C. THEOREM.

If two triangles have the three sides of the one equal to the three sides of the other, each to each, the triangles must be equal in all respects.

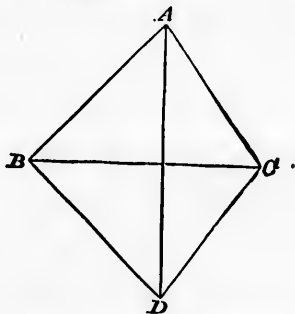


Let the three sides of the Δ s ABC , DEF be equal, each to each, that is, $AB=DE$, $AC=DF$, and $BC=EF$.

Then must the triangles be equal in all respects.

Imagine the ΔDEF to be turned over and applied to the ΔABC , in such a way that EF coincides with BC , and the vertex D falls on the side of BC opposite to the side on which A falls; and join AD .

CASE I. When AD passes through BC .



Then in ΔABD , $\because BD=BA$, $\therefore \angle BAD=\angle BDA$, I. A.

And in ΔACD , $\because CD=CA$, $\therefore \angle CAD=\angle CDA$, I. A.

$\therefore$ sum of $\angle$ s BAD , CAD =sum of $\angle$ s BDA , CDA , Ax. 2.
that is, $\angle BAC=\angle BDC$.

Hence we see, referring to the original triangles, that

$$\angle BAC=\angle EDF.$$

$\therefore$, by Prop. 4, the triangles are equal in all respects.