Mathematical Department.

SOLUTIONS TO ENTRANCE ARITHMETIC.

(See January number, Page 6, for Questions.)

1. Indicating the work first, we have the expression

$$(59404+47675)(59404-47675)\div 7\times 13\times 19,$$
$$=(107079\times 11729)\div 7\times 13\times 19,$$
$$=(15297\times 11729)\div 13\times 19,$$
$$=179019261 3\div 13\times 19,$$

$$13\,\underline{|\,1790192613,}$$
$$19\,\underline{|\,137707124}-1,$$
$$7247743-7,$$

$\therefore$ Quotient $=7247743$, and remainder $=92$.

2. Sold 5 for 11c., or 5 doz. for 132c.
Bought 5 doz. " 50c.
Gain on 5 doz. = 82c.
" 20 " = 328c.
" 11 boxes = \$36.08.

3. Cost of fence $=\$1\frac{1}{2}\times 2(40+65)$
$=\$3\times 105=\$315$.

Cost of land $=\dfrac{40\times 25}{160}\times\$300=\$1875$,

which is *less* than ten times the cost of the fence.

4. If C gets 1 share, A gets 2 shares, B 2 shares $-\$70$,
$\therefore$ 5 shares $-\$70=\$1200$,
1 share $=\$1270\div 5=\$254=C$'s,
$\therefore$ A's $=\$408$, and B's $=\$438$.

5. $\dfrac{\frac{2}{5}\text{ of }8\frac{1}{3}+2\frac{1}{7}\text{ of }5\frac{5}{6}}{\frac{3}{7}\text{ of }3\frac{1}{2}-\frac{1}{2}\text{ of }\frac{1}{3}\text{ of }2\frac{2}{3}}=\dfrac{\frac{10}{3}+\frac{25}{2}}{\frac{3}{2}-\frac{4}{9}}=\dfrac{60+225}{27-8}=\dfrac{285}{19}=15.$

6. $\dfrac{(1\cdot302+3\cdot2589+40\cdot93)\times\cdot00297}{90\cdot09}$
$=45\cdot4909\times\cdot00297\div 90\cdot09,$
$=6\cdot4987\times\cdot003\div 13=\cdot00149962.$

7. Price $=\dfrac{2875-1083}{2000}\times 16\frac{1}{4}$
$=\dfrac{1792\times 65}{8000}=\cdot224\times 65=\$14.56.$

8. A must allow B a start of 1 minute, *i.e.* one sixth of a mile $=293\frac{1}{3}$ yds.

9. Gang can do $\frac{1}{8}$ of work each day; in 5 days $\frac{5}{8}$ work done; $\frac{3}{8}$ work finished by 2 men in 5 days.
$\therefore$ 1st man did $\frac{3}{16}$ work in 5 days.

10. Interest $=\$275.80\times\frac{91}{365}\times\frac{7}{100}=\$4\cdot813.$

ELEMENTARY ALGEBRA.

1. Multiply

$$\begin{array}{l} a+b \\ \underline{c+d} \\ ac+bc \\ \underline{\quad ad+bd} \\ ac+(bc+ad)+bd \end{array}$$

Observe that the product of the two extreme terms, ac and bd, is the same as the product of the two terms within the bracket, bc and ad. Each pair gives $abcd$. Suppose we wish to *un-multiply* such an expression, we have the clew to the method. Examine the case.

2.
$$\begin{array}{l} x+5 \\ \underline{x+4} \\ x^2+5x \\ \underline{\quad 4x+20} \\ x^2+9x+20. \end{array}$$

The product of the outside terms is $20x^2$. It is necessary then to split the $9x$ into two parts whose product shall be $20x^2$. In other words we have to find two numbers whose sum is 9 and their product 20. The only such numbers are 4 and 5. Thus we have,

$x^2+5x+4x+20$
i.e. $x(x+5)+4(x+5)$, or $(x+5)(x+4)$.

3. Consider $8x^2+34xy+21y^2$. We have to find two numbers whose sum is $34xy$ and their product $8\times 21\times x^2y^2$. Expressing 8×21 in prime factors, $2\times 2\times 2\times 3\times 7$, we can easily form two numbers, 2×3 and $2\times 2\times 7$, whose sum is 34. Thus we get

$8x^2+6xy+28xy+21y^2$
$=2x(4x+3y)+7y(4x+3y)=(4x+3y)(2x+7y).$

4. $11a^2-23ab+2b^2$. Product $+22a^2b^2$, sum $-23ab$. The signs show that both are *minus*; they must be -1 and -22. Then $11a^2-22ab-ab+2b^2=11a(a-2b)-b(a-2b)$, and the factors are
$(a-2b)(11a-b)$.

5. $4(x+2)^4-37x^2(x+2)^2+9x^4$.
Product must $=36x^4(x+2)^4$, and sum $=-37x^2(x+2)^2$;
-1 and -36 are plainly the numbers; hence we have,
$4(x+2)^4-36x^2(x+2)^2-x^2(x+2)^2+9x^4$
i.e. $4(x+2)^2\{(x+2)^2-9x^2\}-x^2\{(x-2)^2-9x^2\}$
or $\{(x+2)^2-9x^2\}\{4(x+2)^2-x^2\}$
$=(x+2-3x)(x+2+3x)(2x+4-x)(2x+4+x)$
$=4(1-x)(1+x)(4+x)(4-3x)$.
The solution becomes neater if we write k for $(x+2)^2$.
The given expression then becomes
$4k^2-37kx^2+9x^4=(k-9x^2)(4k-x^2)$.
Now restore the value of k, and we get the same factors.

6. $(12b^2-29bc+15c^2)x^3+(23b^3-31b^2c-9bc^2+15c^3)x^2+(10b^4-6b^2c^2)x$
Set aside the common factor x; factor the $12b^2-29bc+15c^2$ by the preceding method $=(4b-3c)(3b-5c)$; observe that $10b^4-6b^2c^2=2b^2(5b^2-3c^2)$, that $(5b^2-3c^2)(3b-5c)=15b^3-9bc^2-25b^2c+15c^3$, and that $23b^3-31b^2c-$&c. $=(5b^2-3c^2)(3b-5c)+2b^2(4b-3c)$. Now write k for $(4b-3c)$, m for $(3b-5c)$, and y for $5b^2-3c^2$, and the given expression becomes simplified to $kmx^2+(ym-2kb^2)x+2b^2y$. Now take together the first and third, second and fourth terms, taking out the common factors and we get $kx(mx+2b^2)+y(mx+2b^2)=(mx+2b^2)(kx+y)$. Restore the values of m, k, and y, and we have the factors
$x(3bx-5cx+2b^2)(4bx-3cx+5b^2-3c^2)$.

Elegant examples for practice will be found in *McLellan's Teacher's Handbook*, p. 71, and elsewhere, *e. g.*

$6(x^2+xy+y^2)^2+13(x^4+x^2y^2+y^4)-385(x^2-xy+y^2)^2$;
$21(x^2+2xy+2y^2)^2-6(x^2-2xy+2y^2)^2-5(x^4+4y^4)$;

of which the factors are, respectively,
$2(4xy-3x^2-3y^2)(61x^2-49xy+61y^2)$, and
$2(5x^2+4xy+10y^2)(x^2+10xy+2y^2)$.

ONTARIO.—JULY EXAMINATION, 1882.

FIRST CLASS TEACHERS—GRADE C.

EUCLID.

TIME—THREE HOURS.

1. The three angles of a triangle are together equal to two right angles.

If triangles be formed on the sides of a polygon of n sides by producing the alternate sides to meet, the sum of the vertical angles of these triangles is equal to $2n-8$ right angles.

2. Establish the converse of the following: The complements of the parallelograms, which are about the diameter of any parallelogram, are equal to one another.

3. To divide a given straight line into two parts, so that the rectangle contained by the whole and one part may be equal to the square on the other part.

Point out all the lines in the figure that are divided similarly to the given line.

4. By the assistance of Prop. 12, Bk. II., when the sides of a triangle are 25, 45, and $20\sqrt{10}$ find its area.

5. If in a circle all possible chords be drawn passing through the same point in the circumference, and these chords be doubled in length by production, the locus of the extremities of the lines so formed is a circle.

6. The angles in the same segment of a circle are equal to one another.

If a line of constant length move with its extremities in two fixed lines, and at the ends of the first line lines be drawn perpendicular to the two fixed lines, the locus of the intersection of these lines is a circle.

7. ABC is a triangle, C being a right angle. On CA, CB are described segments of circles containing angles equal to CBA, CAB respectively. Show that the circles of which these segments are parts touch one another.