## ARTS DEPARTMENT.

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Our correspondents will please bear in mind, that the arranging of the matter for the printer is greatly facilitated when they kindly write out their contributions, intended for insertion, on one side of the paper ONLY, or so that each distinct answer or subject may admit of an easy separation from other matter without the necessity of having it re-written.

Problems have been received from J. H BALDERSON, B.A., Mathematical Master, High School, Mount Forest, which will appear in our next issue.

### SOLUTIONS

Of Nos. 132, 135, 137 and 138 have been received from D. F. H. WILKINS, B A., Math. Master, High School, Chatham.

Solutions by proposer, ANGUS MACMURCHY, University College.

132. Prove that

$$\log_e \sqrt[4]{x} = (x^{\frac{1}{4}} - 1) \cdot \frac{2}{x^{\frac{1}{8}}+1} \cdot \frac{2}{x^{\frac{1}{16}}+1} \ldots \text{ad inf.}$$

We have $\log_e x$

$$= (x-1) \cdot \frac{2}{x^{\frac{1}{2}}+1} \cdot \frac{2}{x^{\frac{1}{4}}+1} \cdot \frac{2}{x^{\frac{1}{8}}+1} \ldots \text{ ad inf.}$$

$$= \text{lt. } 2^n \left(x^{\frac{1}{2^n}} - 1\right), \quad n = \infty.$$

$$= \text{lt.}_{a=0} \left(\frac{x^a - 1}{a}\right).$$

$$= \log_e x.$$

or $\log_e x = 4(x^{\frac{1}{4}} - 1) \dfrac{2}{x^{\frac{1}{8}}+1} \ldots$ ad inf.

*i.e.*, $\log_e x^{\frac{1}{4}} = (x^{\frac{1}{4}} - 1) \dfrac{2}{x^{\frac{1}{8}}+1} \cdot \dfrac{2}{x^{\frac{1}{16}}+1}$ ..ad inf.

133. Prove that

$$\frac{2^{r-1}-1}{\lfloor r} = \frac{1}{\lfloor r-1 \, \lfloor 1} + \frac{1}{\lfloor r-2 \, \lfloor 2} + \ldots$$

$$+ \frac{1}{2\left\{\left\lfloor \frac{r}{2} \right.\right\}^2}, \text{ or } \frac{1}{\left\lfloor \frac{r+1}{2} \right. \left\lfloor \frac{r-1}{2} \right.}$$

according as $r$ is even or odd.

We have $e^{2x} = e^x \times e^x$

$$= 1 + \frac{2x}{\lfloor 1} + \frac{(2x)^2}{\lfloor 2} + \ldots + \frac{(2x)^r}{\lfloor r} + \ldots$$

$$e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \ldots + \frac{x^r}{\lfloor r} + \ldots$$

Picking out coefficient of $x^r$ in $e^{2x}$ and $e^x \times e^x$, we have

$$\frac{2^r}{\lfloor r} = 2\left\{\frac{1}{\lfloor r} + \frac{1}{\lfloor r-1 \, \lfloor 1} + \ldots\right\}.$$

Divide through by 2 and transpose first term of dexter.

134. Prove that $\dfrac{2^{2n}-1}{\lfloor 2n+1}$

$$= \frac{1}{\lfloor 2n-1 \, \lfloor 2} + \frac{1}{\lfloor 2n-3 \, \lfloor 4} + \ldots + \frac{1}{\lfloor 2n}.$$

Substituting, the series in terms of circular measure of angle, for $\sin 2\theta$, $\sin \theta$, and $\cos \theta$ in formula $\sin 2\theta = 2 \sin \theta \cos \theta$, and equating coefficients of $\theta^{2n+1}$, we get result stated above.

135. In any plane triangle $ABC$, if $\Sigma \sin^2 A = 2\sigma$, and if $\sigma - \sin^2 A = \sigma_1$, &c., then

$$\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 = \sin^2 A \sin^2 B \sin^2 C.$$

$$\sigma_1 = \frac{\sin^2 B + \sin^2 C - \sin^2 A}{2}$$

$$= 2\Delta^2(b^2 + c^2 - a^2).$$

($\Delta$ = area of triangle $ABC$)

&c. = &c.