from the key as to bring the point J to the centre of CD and eventually even to the extrados. We have been assuming, however, that the loading acts vertically; but in such a case the assumption may reach a limit beyond which it is no longer true. When the loading is so highly concentrated over the haunches it is more than probable that the direction of the pressure becomes inclined to the back of the arch ring. This is difficult to estimate in amount, but in semi-circular or elliptical arches it will be quite sufficiently allowed for by supposing that when J reaches the centre of CD, the line LJ then becomes tangent to the centre line of the arch ring. In a segmental arch in like manner, the pressure of the spandrel will prevent J from passing the center of CD. It would, however, be preferable to increase the thickness of the arch ring towards the springing, or to redistribute the weight in the spandrel, rather than to count upon any inclination in the pressure above the joint of rupture to improve the position of the curve of pressure. Such positions of J are not likely to occur for any ordinary form of spandrel.

Let us now compare this method with the theory that the curve of pressure must remain within the middle third of the arch ring. This theory is endorsed by Rankine, and the weight of his authority has tended more than it should to deter further investigation. His statement is that the stability of an arch is secure if the curve of pressure can be drawn within the middle third, and he goes so far as to say that although arches have stood and still stand in which the curve lies beyond the middle third, the stability of such arches is either now precarious or must have been precarious while the mortar was fresh. (13) We must first endeavor to ascertain the nature of the arch and the conditions of which Rankine is treating. It is to be inferred from the cases he takes up and the examples he gives, that he is considering arches for which the amount of the moving load can be neglected in comparison with the weight of the structure itself. Prof. Wm. Allan in his "Theory of Arches," which he describes as being an amplification and explanation of Rankine's chapters on the subject, takes for granted that this is Rankine's point of view; "In all stone or brick arches the changes in the curve of pressure due to passing loads are usually slight, because the weight of such passing loads is generally small compared with the weight of the arch itself and its backing." (14) This statement affords the key to Rankine's explanation of the subject. If this had been distinctly pointed out in his works, a large amount of discussion and misunderstanding might have been avoided.

The same explanation of the range of application of the theory of the middle third is given in the article on "Bridges" in the last edition of the *Encyclopædia Britannica*: "The masonry arch differs from the superstructure of other bridges in the following respect: it depends for its stability on the presence of a permanent load specially arranged, and so considerable in amount that the changes produced in the direction and magnitude of the stresses by the passing load are insignificant." (15)

The theory of the middle third corresponds then to the case of the finished structure without appreciable moving load. To find in accordance with the method suggested above the limits within which the gravity line must keep in order that the curve should remain within the middle third of the arch ring, is merely a matter of geometrical construction; and we have thus a ready means of comparison by which to verify the results. Take for example the case of a semi-circular arch of radius r, in which the arch ring has a uniform thickness t, and the joint of rupture is at 30°. The possible limiting positions of K and J for a curve remaining in the middle third will be, the upper third at the key and the lower third at the joint of rupture, or the lower third at the key and the upper third at the joint of rupture. The distance y of the gravity line from the key corresponding to these limits will be: $y = r \tan 30°$ and $y = (r + t) \tan 30°$. Or numerically, for semi-circular arches of 10 ft. and 100 ft. diameter, in which the thickness t is determined by Rankine's formula, the limits will be:—

10 ft. arch 0.577 r and 0.664 r. 100 ft. arch, 0.577 r and 0.605 r.

As it happens, the gravity line for spandrels as generally built has very nearly this position in full arches and a corresponding one in segmental arches; and this accounts for the currency of the theory as applied to the conditions supposed.

This theory also implies, as we have seen, that every joint in the arch must be entirely in compression. This corresponds with a much higher degree of stability than is necessary under quiescent loads; but the advocates of the theory take it practically as a margin of safety. They consider that if the curve is within the middle third in the structure bearing its own weight, it will be sufficiently stable under any moving loads it may be called upon to carry. (16) As the theory is fairly in accord with the form which the spandrel usually has, it may be sufficiently near the truth as regards road bridges; but it is taking quite too much for granted in the case of arches carrying heavy engine loads. It could only be brought into reasonable agreement with such cases by lowering the springing and increasing the depth and weight of the spandrel, till the moving load became relatively small enough to neglect; but such a construction would be inconsistent with economy. To have the curve of pressure within the middle third of the arch ring is very desirable and should be aimed at; but it cannot be laid down as a