Values.

20 10. Ascertain the cost, at \$35.10 per ton of 2000 lbs., of 864 yards of iron piping, 25 inches internal diameter, and half an inch thick, assuming the specific gravity of iron to be 7·77, and a cubic foot of water to weigh $62\frac{1}{2}$ lbs. ($\pi = 3\frac{1}{7}$).

SOLUTIONS.

1. £8 15*s.* 7*d.*

2. ·03749; ·012602.

3. ·311768; $= \frac{3\sqrt[3]{\cdot 02} - 23 \times \cdot 125\sqrt[3]{\cdot 02}}{2\sqrt[3]{\cdot 02} + \sqrt[3]{\cdot 02}} = \frac{3 - 2\cdot 875}{3} = \cdot 41\dot{6}$.

4. Area of path = 6174 sq. ft.; of rest, 18126. Cost = $\frac{6174}{9} \times \cdot 22\frac{1}{2} + \frac{18126}{9} \times \cdot 17\frac{1}{2} = 154\cdot 35 + 31\cdot 7205 = \$186\cdot 07$.

5. Interest for $2\frac{1}{2}$ years = \$693.88 − \$640.80½ = \$52.52½.

∴ int. for 1 yr. = \$21.01; ∴ principal = 693.88 − 21.01×8 = \$525.25.

Also, 21.01 on 525.25 is $\frac{21.01}{525.25}$ on 1, or $\frac{2101}{525.25}$ on 100; *i.e.*, 4%.

6. Cost price of mixture must be $\frac{10}{11}$ of 2.80 = $\$2.54\frac{6}{11}$. Hence gain on each gallon of the cheap kind was $14\frac{6}{11}$ cts., and loss on each gallon of the dear kind was $65\frac{5}{11}$ cts. The gain in one case must be counterbalanced by the loss in the other; hence they must be mixed in ratio $\frac{65\frac{5}{11}}{14\frac{6}{11}} = 9:2$.

7. Gain on 112 through advance in stock was $3\frac{3}{4}$; interest on 112 was 4, making a total profit on 112 of $7\frac{3}{4}$, or 1 on $\frac{112}{7\frac{3}{4}}$, or 310 on $\frac{112}{7\frac{3}{4}} \times 310$, or 310 on \$4480.

8. Let 5 and 8 represent their capitals. Then

$$A\text{'s stock} \begin{cases} 5 \text{ for } 6\frac{1}{2} \text{ months} = 32\frac{1}{2} \text{ for 1 month.} \\ 4 \text{ for } 5\frac{1}{2} \quad \text{"} \quad = 22 \quad \text{"} \quad \text{"} \end{cases}$$

$$\overline{54\frac{1}{2}} \quad \text{"} \quad \text{"}$$

$$B\text{'s stock} \begin{cases} 8 \text{ for } 7\frac{1}{2} \text{ months} = 60 \text{ for 1 month.} \\ 5\frac{1}{3} \text{ for } 4\frac{1}{2} \quad \text{"} \quad = 24 \quad \text{"} \quad \text{"} \end{cases}$$

$$\overline{84}$$

A's gain = $\frac{84}{54\frac{1}{2} + 84}$ of 3047 = \$1848; ∴ B's = \$1199.

9. To gain 10 per cent. per annum the broker's \$375.80 must become, at the end of the 75 days, 375.80 + $\frac{75}{36500}$ of 10 of 375.80 = \$383.52. And the present value of this for 135 days at 8 per cent. = $\frac{100}{102\frac{78}{73}}$ of 383.52 = \$372.50 +.

10. Value = $\frac{\{(13)^2 - (12\frac{1}{2})^2\}3\frac{1}{7}}{144} \times 864 \times 3 \times 62\frac{1}{2} \times 7.77 \times \frac{35.10}{2000}$ = \$6147.31.

ALGEBRA.

TIME—TWO HOURS.

Examiner—J. C. GLASHAN.

Values.

8 1. Find the value of
$3x^5 + 54x^4 + 50x^3 - 19x^2 - 35x - 18$, when $x = -17$.

2. Demonstrate the identities:

8 (*a*) $(5m^2 + 4mn + n^2)^2 - (3m^2 + 4mn + n^2)^2 = 4m^2(2m + n)^2$.

8 (*b*) $(a+b+c)(ab+bc+ca) - abc = (a+b)(b+c)(c+a)$.

8 (*c*) $(a-b)(c-d) + (b-c)(a-d) + (c-a)(b-d) = 0$.

8 3. Divide $(m^2 + an^2)(x^2 + ay^2) - a(nx - my)^2$ by $mx + any$.

Values.

8 4. Prove that if from the square of the sum of two numbers there be taken four times their product, the remainder is a square.

5. Solve

8 (*a*) $(x-1)(x-2) - (x-3)(x-4) = 3$.

8 (*b*) $\frac{2}{x-1} + \frac{3}{x-2} = \frac{8}{x^2 - 3x + 2}$.

8 (*c*) $(x-a)(b-c) + (x-b)(c-a) + (x-c)(a-b) = x - a - b - c$.

10 6. What value of x will make $x^2 + 2ax + b^2$ the square of

8 $x + c$? What is the result when $a = b = c$?

10 7. A man is thrice as old as his son, five years ago he was four times as old; how old is he?

SOLUTIONS.

1. Dividing by $x + 17$, we see the expression equals $(x + 17)(3x^4 + 3x^3 - x^2 - 2x - 1) - 1$, and when $x = -17$, the first factor equals zero, and expression becomes −1.

2. (*a*) = $\left\{ m^2 + (2m+n)^2 \right\}^2 - \left\{ -m^2 + (2m+n)^2 \right\}^2 = 4m^2(2m+n)^2$.

(*b*) If $-b$ be written for a in the left hand side, it vanishes, so that $a + b$ is a factor of that side; and then by symmetry $b + c$ and $c + a$ must also be factors. This side being of three dimensions must therefore equal $K(a+b)(b+c)(c+a)$, where K is some quantity independent of a, b and c. To find it let $a = b = c = 1$; ∴ $K = 1$, and identity is established.

(*c*) Putting $a = 0$ in the left hand side, it vanishes. And from the symmetrical way in which a, b and c are involved, it would therefore vanish for $b = 0$ and $c = 0$. Hence abc would appear to be a factor; but this is impossible since the expression is of only two dimensions. It must therefore vanish for all values of the letters involved, *i.e.*, it is identically equal to zero.

3. Dividend $= m^2x^2 + a(n^2x^2 + m^2y^2) + a^2n^2y^2 - a(n^2x^2 + m^2y^2) + 2amnxy = (mx + any)^2$; ∴ quotient $= mx + any$.

4. Let a, b be the numbers. Then $(a+b)^2 - 4ab = (a-b)^2$, a square.

5. (*a*), $3\frac{1}{2}$. (*b*), 3. (*c*). The left hand member of the equation is evidently identically equal to zero, being in fact the same as (*c*) in question 2, with x for d. Hence equation becomes $0 = x - a - b - c$, or $x = a + b + c$.

6. $x^2 + 2ax + b^2 = x^2 + 2cx + c^2$, or $x = \frac{b^2 - c^2}{2(c-a)}$. If $a = b = c$, x assumes the indeterminate form $\frac{0}{0}$, *i. e.*, x may have any value, the two expressions $x^2 + 2ax + b^2$, $x^2 + 2cx + c^2$ being identical, and therefore equal for all values of x.

7. If x be present age of father, equation is $x - 5 = 4\left(\frac{x}{3} - 5\right)$; ∴ $x = 45$.

EUCLID.

TIME—TWO HOURS.

Examiner—JOHN J. TILLEY.

N.B.—*Eight questions to count a full paper; value,* $12\frac{1}{2}$ *for each.*

1. (*a*) Define Scalene Triangle, Point, Straight Line, Square, and distinguish between Problem and Theorem, Direct and Indirect demonstrations.

(*b*) What propositions in Euclid, Book I, are proved by the latter method?

2. If one side of a triangle be produced the exterior angle is greater than either of the interior opposite angles. Give full proof for one exterior angle.