ARTS DEPARTMENT.

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SOLUTIONS.

42. ABC is any $\triangle$ inscribed in a $\bigcirc$ $ABEC$, of which AE is a diameter. AQ bisects $\angle$ between AE and the $\perp$ from A on base BC; shew $\angle BAC = \angle$ subtended by arc CQ at centre of $\bigcirc$.

Take centre of $\bigcirc =$

$$\angle QOE = 2\angle QAE = 2\angle QAD$$
$$= \angle QAE + \angle QAD = \angle DAE.$$
$$\angle EOC = 2\angle OAC.$$

Join EC; then $\angle ECA$ is a rt. $\angle$, and $\angle AEC = \angle CBA$;

$\therefore$ $\triangle$s EAC, ABD are equiangular, *i.e.*, $\angle BAD = \angle EAC$.

But $\angle EOC = 2\angle EAC$;

$\therefore$ $\angle EOC = \angle EAC + \angle BAD$.

Hence $\angle QOC = \angle BAC$.

45. Let it be required to inscribe in ABC a $\triangle$ similar to DEF.

Let ABC and DEF be so placed in the same plane that no side of the one may be parallel to any side of the other. Describe $\triangle$ NKM round DEF by drawing through D, KM parallel to AC, etc. Then if we divide AC, AB, BC at R, S and T proportionally to DK, DM, KF, NF, etc., and join SRT, it is easy to shew $\triangle$ SRT is similar to FDE.

52. Prove analytically (1) angle in a semicircle is a right angle; (2) angles in the same segment of a circle are equal.

(1) Let APB be a semicircle, AB a diameter the axis of x, A origin, P any point in circumference.

$y^2 = 2ax - x^2$, equation to circle.

Let $y = mx$ be equation to AP; then at point P, $x = \frac{2a}{1+m^2} y = \frac{2am}{1+m^2}$,

$\therefore$ equation to BP is $y = -\frac{1}{m}(x - 2a)$.

$\therefore$ BP is perpendicular to AP.

(2) Same figure as (1).

Let equation to circle be $x^2 + y^2 = ax + by$. A origin, then $AB = a$. Let P be any point $x'y'$ in circumference.

$$\tan APB = -\tan(PAB + PBA)$$
$$= -\frac{\left(\frac{y'}{x'} + \frac{y'}{a-x'}\right)}{1 - \frac{y'^2}{x(a-x')}} = \frac{a}{b}.$$

$\therefore$ angle $APB = \tan^{-1}\frac{a}{b}$, which is independent of position of P.

53. An ellipse and hyperbola that have the same foci and centre, will cut one another at right angles.

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be equation to ellipse,

and $\frac{x^2}{a'^2} - \frac{y^2}{b'^2} = 1$ be equation to hyperbola,

$\therefore$ $CS^2 = a^2 - b^2 = a'^2 + b'^2$.

At points of intersection

$$\left(\frac{1}{a^2} - \frac{1}{a'^2}\right)x^2 + \left(\frac{1}{b^2} - \frac{1}{b'^2}\right)y^2 = 0$$
$$\therefore \frac{x^2}{y^2} = \left(\frac{aa'}{bb'}\right)^2.$$

Now, if θ, θ' be angles which tangents at points of intersection make, with axis of x,

$$\tan\theta \tan\theta' = -\left(\frac{bb'x}{aa'y}\right)^2$$

$\therefore$ $\tan\theta \tan\theta' = -1$, or tangents at points of intersection are at right angles.