Also from same formula $\frac{n}{m}=\frac{m}{n}+(\text{no. of terms}-1)\,\frac{n-m}{mn}$ giving no. of terms$=m+n+1$

$\therefore s=\left\{2\frac{m}{n}+(m+n)\frac{n-m}{mn}\right\}\frac{m-n+1}{2}=$given expression.

9. $r=\frac{1}{\sqrt{2}}$; $\therefore S=\frac{8}{\sqrt{3}}\frac{\left(\frac{1}{\sqrt{2}}\right)^{3n}-1}{\frac{1}{\sqrt{2}}-1}=\sqrt{3}\,\frac{\frac{1}{2^n}-1}{\frac{1}{\sqrt{2}}-1}=$

$$\text{And } \Sigma=\frac{\sqrt{3}}{1-\frac{1}{\sqrt{2}}}=\frac{\sqrt{6}}{\sqrt{2}-1}.$$

10. (1) $\frac{|\underline{7}}{|\underline{2}\ |\underline{3}}$. (2). By Todhunter, § 501, the number of permutations of 10 things where each may occur once, twice .... six times is 10^6. From these we are to exclude the ones in which a particular digit occurs but once, of which there are $\frac{|\underline{10}}{|\underline{4}}$, making $10^6-\frac{|\underline{10}}{|\underline{4}}$ in all. But we must also exclude those numbers in which ciphers occur to the left. Thus the number in which one cipher occurs to the left is $10^5-\frac{|\underline{10}}{|\underline{5}}$; in which two ciphers occur $10^4-\frac{|\underline{10}}{|\underline{6}}$, &c.

11. $+\frac{\frac{3}{2}\left(\frac{3}{2}-1\right)\ldots\ldots\left(\frac{3}{2}-5\right)}{|\underline{6}}x^3.$

$$n^{\frac{n}{1-n}}=n^{-1}\,.\,n^{\frac{n}{n-1}}=n^{-1}\left\{1-(1-n)\right\}^{\frac{1}{1-n}}$$
$$=n^{-1}\left\{1-1+\frac{n}{|\underline{2}}-\frac{n(2n-1)}{|\underline{3}}+\&c.\right\}$$
$$=\frac{1}{|\underline{2}}-\frac{2n-1}{|\underline{3}}+\&c.$$

TRIGONOMETRY—HONORS.

Examiner: A. K. Blackadar, B.A.

1. Define the common logarithm of a number.
If x be the logarithm of N to base 2, and 41·664 be the logarithm of N to base 8; find the common logarithm of x.

2. Prove
(1) $\log\frac{ab}{c}=\log a+\log b-\log c$.
(2) $\log\sqrt[n]{a^m}=\frac{m}{n}\log a$.
(3) Find Log cos 30°.

3. Perform the following operations by means of logarithms:
(1) Divide 416·64 by $\sqrt{623640}$.
(2) Find the value of $\frac{(\cdot 25)^{-5}\times\sqrt[3]{\cdot 072}}{(527\cdot 58)^{10}}$

4. Having given
$L\sin 28°21'=9\cdot 676582$,
Difference for $1'=234$,
$L\tan 61°39'=10\cdot 267952$,
Difference for $1'=302$,
Find (1) $L\cos 28°\,21'\,20''$; (2) $L\sin 123°\,18'\,30''$;
(3) the angle the Log of whose secant is 10·055469.

5. Prove
(1) $\tan A=\sqrt{\sec^2 A-1}$;
(2) $\cos\theta=\cos(2n\pi\pm\theta)$.
(3) $\cos^2\theta-\tan^2\theta=\frac{4}{\tan 2\theta\sin 2\theta}$.

6. Prove the following, when $A+B$ is less than 90°, and without assuming the formulæ for $\sin(A+B)$ and $\cos(A+B)$:
(1) $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$.
(2) $\cos 2A=1-2\sin^2 A$.

7. If $\tan A$ and $\tan B$ be the roots of the equation $x^2-4nx+1=3n$, shew that
$A+B=2\tan^{-1}2^{-1}$ or $=2\tan^{-1}(-2)$.

8. In any triangle ABC, prove the following formulæ:
(1) $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$.
(2) $\tan\frac{1}{2}A=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$.

If AD bisects the angle A, and AE is drawn perpendicular to the base BC, shew that
$$\cos DAE=\frac{b+c}{a}\sin\tfrac{1}{2}A.$$

9. Having given
(1) $b=103\cdot 5$, $c=520\cdot 14$, $C=90°$, solve the triangle.
(2) $a=388\cdot 88$, $b=189\cdot 20$, $C=91°\,48'$, find A, B and c.

10. If $\left.\begin{array}{l}\sin\beta-\cos\alpha=a\\ \sin\alpha-\cos\beta=b\end{array}\right\}$
shew that $\tan\frac{1}{2}(a-\beta)=\frac{a-b}{a+b}$,
and $\sin(a-\beta)=\frac{a^2-b^2}{a^2+b^2}$.

11. A person finds the elevation of the *bottom* of a flagstaff on a tower to be 30°; receding 60 feet up a hill, which is inclined 24° to the horizon, he finds the elevation of the *top* of the staff to be 30°; shew that the length of the flagstaff is 56·050 feet.

Number.	Log.	Angle.	Log.
20000	30103	tan 78° 31′	10·69241
30000	47712	sin 88° 12′	9·99979
62364	79494	cosec 68° 42′	10·03071
41664	61976	tan 24° 36′	9·66067
24918	39651	tan 44° 6′	9·98645
50974	70735		
10350	01494		
52758	72229		
38838	·58926		

SOLUTIONS.

1. $2^x=N=8^{41.664}=2^{41.664\times 3}$; $\therefore x=41.674\times 3$, and $\log x=2\cdot 09688$.

2. (1) and (2) book-work. (3). $=\frac{1}{2}\log.3-\log 2+10=9.93753$.

3. (1) ·52758.
(2) $=\frac{\left(\frac{1}{2^2}\right)^{-5}\times\left(\frac{2^3\times 3^2}{10^3}\right)^{\frac{1}{3}}}{(527.58)^{10}}=\frac{2^{11}\times 3^{\frac{2}{3}}}{(527.50)^{10}\times 10}$, the log. of which appears to be $\overline{25}\cdot 40651$, the number corresponding to which is not given, 39651 having evidently been the logarithm obtained.