

$= 2x$. Curve bisects sub-tangent. Length of normal $= 2d$. Analytical investigation into diameters and their properties (alternative with § 147.)

Geometrical proof of the equation to the parabola referred to diameter and tangent, together with a proof that the chords parallel to the tangent are bisected, &c., (as in the obligatory course.

To draw a parabola, given any diameter and the tangent at its vertex and one other point.

To draw a parabola touching two intersecting straight lines at given points; also, to construct the focus and directrix, the latter by at least six points.

To draw a parabola, given its vertex, axis and one point; thence to draw it, given the axis and two points at different distances from the axis.

Construction of tangents from any external point; their lengths are proportional to the cosecants of their inclinations.

Intersections of Conics, straight lines and other curves.

Contact. Circle of curvature; 2ρ as limit of $\frac{y^2}{x}$ or $\frac{y^2}{x \sin \theta}$

$$\therefore \rho = \frac{2a}{\sin^3 \theta} = \frac{N}{\sin^2 \theta} = \frac{N^2}{SL^2}; \text{ thence construction of radius of curvature, and evolute.}$$

Intersection of circle and conic, equal inclination of opposite chords; thence construction of radius of curvature, § 208.

Ellipse.—Chapter IX, X, omitting § 205.

Equation found from the definitions of an ellipse as the projection of a circle, as described by the trammel, and as $r + r' = 2a$, instead of that given in Todhunter. Geometric properties proved from the definition $r + r' = 2a$, as follows: Construction of a tangent; its equal inclinations to the focal distances; locus of the foot of the perpendicular from the focus.

$$pp' = b^2; \frac{p}{p'} = \frac{r}{r'}; p^2 = \frac{b^2 r}{r'}$$

Locus of intersection of tangent with the perpendicular at the focus to the radius vector; proof of Todhunter's definition of an ellipse; locus of intersection of tangent at the ex-

tremities of a focal chord; straight lines ae, a_e ; $r = a \pm ex$.

Polar equation referred to both focus and centre. Equations to tangent and normal. Points where they cut the axes. The length $e^2 x'$ both analytically and geometrically.

Equation at the vertex becomes a parabola if $e = 1$ or $a = \infty$. Latus rectum $= 2 \frac{b^2}{a} = 2e \left(\frac{a}{e} - ae \right)$, compared with