vation and horizontally from a maximum in the middle, or the point where the deflection is a maximum towards the abutments. The thickening of the dam in the middle to take care of cantilever stress also stiffens the arch materially, considering it as a curved beam. It acts as such to a large extent towards the foundation where t is large compared with R_u.

Before attempting to find what proportion of the load is carried by the arch and what proportion is carried by the cantilever it must be determined how much of the total load is carried by the initial stresses in the arch.

By initial stresses are meant stresses principally due to the weight of the structure and to the water pressure. Therefore, these stresses reach their maximum values at or near the foundation and are zero at the crest. They have not been much discussed so far, but are very important and should be taken into consideration when attempting to find the actual division of load between arch and cantilever. When a body is compressed the dimension in the direction of the compressive force becomes smaller, but in other directions the body swells if free to move (lateral strain). The ratio of lateral to longitudinal strain for concrete has been taken $\frac{1}{m} = \frac{1}{5}$ in the following calculations.*

Any horizontal layer of material will have to sustain compression corresponding to the height of masonry above it and will therefore actually become shorter in a vertical direction and have a tendency to expand horizontally. If the abutments are unyielding the arch may be prevented from actually becoming longer, in which case axial compression is introduced the same as if water pressure acted upon the structure.

If the specific gravity of the concrete for the dam is taken at 2·3 and the height of the dam H, then the average vertical pressure can be expressed as $\frac{2 \cdot 3\,H}{a}$; where a is the ratio of total height of dam to height of a rectangular wall having the same sectional area and the same base. The ratio a is known as soon as the section is known, and in dam design the section must be more or less determined before final calculation can be made.

The dam section in Fig. 3 has an area of 9,668 feet, a base width of 70 feet, and a height of 250 feet. The height of masonry column causing the mean vertical pressure is, therefore, $\frac{9668}{70} = 138$ feet and $a = \frac{250}{138} = 1 \cdot 81$; making the mean vertical compression upon the foundation in terms of head of water equal to $\frac{2 \cdot 3\,H}{1 \cdot 81} = 1 \cdot 27\,H$, with no water pressure upon the upstream side.

The condition of full reservoir introduces an additional force, viz., the radial water pressure, tending to compress the dam body in a direction perpendicular to the direction of the compressive force due to the weight of the body. At the bottom of the dam this force is equal to H in case the water is standing to the crest of the dam. In this case the radial water pressure tends to counteract the swelling of concrete in an up and downstream direction (due to the weight) thereby introducing additional initial axial compression. The total resulting initial axial compression at the foundation of section shown in Fig. 3 is therefore (in terms of head of water):

$$1/5\,(1 \cdot 27\,H + H) = 0 \cdot 454\,H \quad - \quad (5)$$

where Poisson's ratio has been taken equal to 1/5.*

The height of water, h, that this initial axial compression of $0 \cdot 454\,H$ will resist without causing any shortening of the length of the arch at the bottom can be found by using (1), thus:

$$h = 0 \cdot 454\,H \frac{t}{R_u} \quad - \quad - \quad - \quad (6)$$

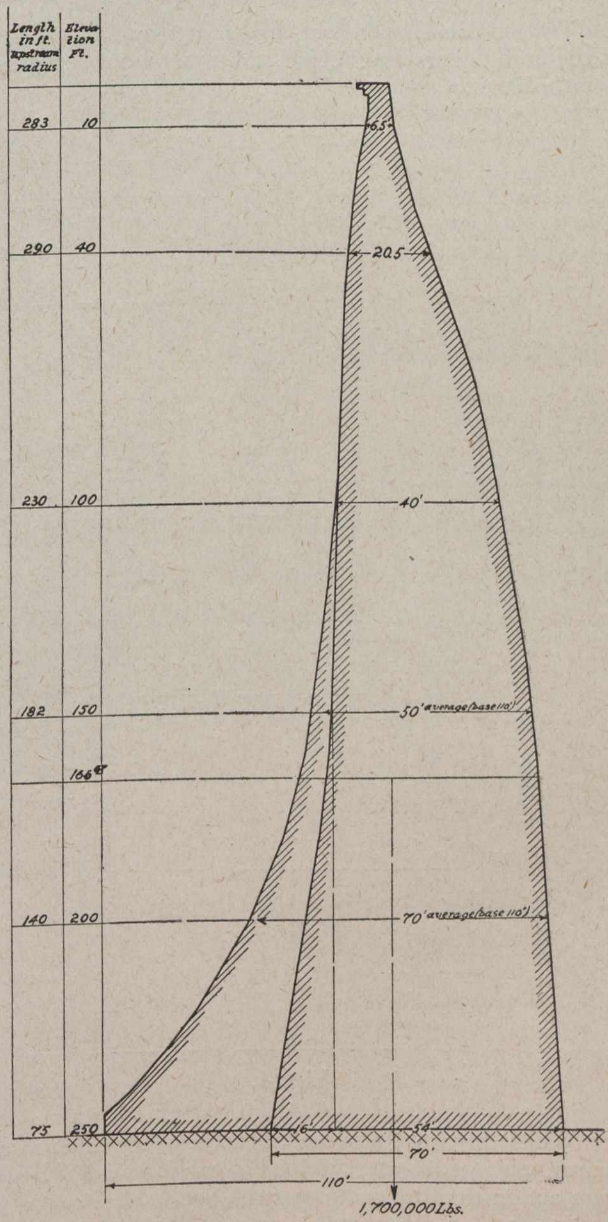

Fig. 3.

For the narrower section, shown in Fig. 3, $t = 70$ feet at the base and $R_u = 75$ feet. Substituting these values, it is seen that this section at the very bottom is able to carry $h = 0 \cdot 454 \times H \times \frac{70}{75} = 0 \cdot 425\,H$, or 42·5 per cent. of the total head of water as an arch before any shortening in the length of the arch occurs.

*Prof. C. von Bach has been kind enough to make some tests for the writer to determine m for concrete 1 : 2 : 3. He found for specimens 45 days old, using between 0.1 and 24 kg. per sq. cm. compression, m to be 5.3. See also Bach's Elastizitat und Festigkeit, 5 Auflage Seite 301. Considering that large stone will be embedded in the concrete in most dams the factor 5 has been used for m as probably representing most closely actual conditions.

*Mr. H. Ballet was probably the first to point out the necessity of taking Poisson's ratio into consideration when attemping to find the actual stresses in a dam body.—Proceedings of the Institute of Civil Engineers, 1909, Page 51.