(3). $\frac{66x+1}{1.5x+1} + \frac{4x+5}{.5x-1} = 52$

divide both sides by 2; then

$$\frac{66x+1}{3x+2} + \frac{4x+5}{x-2} = 26$$

$$22 - \frac{43}{3x+2} + 4 + \frac{13}{x-2} = 26$$

$$\frac{43}{3x+2} = \frac{13}{x-2}$$

$$x = 28$$

(4). $\frac{\sqrt{a} - \sqrt{a - \sqrt{a^2 - ax}}}{\sqrt{a} + \sqrt{a - \sqrt{a^2 - ax}}} = b$ (1)

Subtract 1 from both sides, and

$$\frac{-2\sqrt{a - \sqrt{a^2 - ax}}}{\sqrt{a} + \sqrt{a - \sqrt{a^2 - ax}}} = -b1 \text{ (2)}$$

Add 1 to each side of (1), and

$$\frac{2\sqrt{a}}{\sqrt{a} + \sqrt{a - \sqrt{a^2 - ax}}} = b + 1, \text{ (3)}$$

divide (2) by (3) and we get

$$-\frac{\sqrt{a - \sqrt{a^2 - ax}}}{\sqrt{a}} = \frac{b-1}{b+1}$$

squaring both sides gives

$$\frac{a - \sqrt{a^2 - ax}}{a} = \left(\frac{b-1}{b+1}\right)^2$$

$$1 - \frac{\sqrt{a^2 - ax}}{a} = \frac{(b-1)^2}{(b+1)^2}$$

$$\frac{\sqrt{a^2 - ax}}{a} = 1 - \frac{(b-1)^2}{(b+1)^2}$$

$$\frac{\sqrt{a - x}}{\sqrt{a}} = \frac{4b}{(b+1)^2}$$

$$\frac{a-x}{a} = \frac{16b^2}{(b+1)^2}$$

$$\therefore x = a\left\{1 - \frac{16b^2}{(b+1)^2}\right\}$$

8. (1). $\frac{4}{15}$. (2). The two together can do $\frac{1}{m}$ of the work in one day and $\therefore$ $\frac{n}{m}$ in n days so that when A is called off there remains $\frac{m-n}{m}$ of the work to be done. B does this in p days, and consequently does the whole work in $\frac{mp}{m-n}$ days. In one day $\therefore$ B can do $\frac{m-n}{mp}$ of the work, and the two together can do $\frac{1}{m}$ of it, $\therefore$ A can do $\frac{1}{m} - \frac{m-n}{mp}$ of the work in one day, and consequently does the whole work in $\frac{mp}{p-m+n}$ days.

9. (1) $(x+22)^{\frac{1}{3}} - (x+3)^{\frac{1}{3}} = 1$

cubing both sides, bearing in mind that

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$, gives

$(x+22) - (x+3) - 3(x+22)^{\frac{1}{3}}(x+3)^{\frac{1}{3}} = 1$

$\therefore (x+22)^{\frac{1}{3}}(x+3)^{\frac{1}{3}} = 6$

$\therefore (x+22)(x+3) = 216$

$\therefore x = 5$ or -30.

2. $\frac{(a-x)^4 - (x-b)^4}{(a-x) - (x-b)} = \frac{(a-b)c}{(a-x)(-b)}$ (1)

Also, $(a-x) + (x-b) = a-b$ (2)

dividing (1) by (2) gives

$$\frac{(a-x)^4 - (x-b)^4}{(a-x)^2 - (x-b)^2} = \frac{c}{(a-x)(x-b)}$$

$$\therefore (a-x)^2 + (x-b)^2 = \frac{c}{(a-x)(x-b)}$$

Denote $a-x$ by y and $x-b$ by z

$$\therefore y^2 + z^2 = \frac{c}{yx} \text{ and } y + z = a - b$$

$$\because (y+z)^2 = \frac{c}{yz} + 2yx \text{ and } = (a-b)^2$$

$$\therefore yz = \frac{(a-b)^2 \pm \sqrt{(a-b)^4 - 8c}}{4}$$

but $yz = (a-x)(x-b)$

$$\therefore x = \tfrac{1}{2}\left\{a + b + \sqrt[4]{(a-b)^4 - 8c}\right\}$$

3. $x + y + xy = 11$

$\therefore x + y = 11 - xy$

also, $xy(x+y) = 30$

$\therefore xy(11 - xy) = 30$, $\therefore xy = 5$ or 6

$\therefore x + y = 6$ or 5, $\therefore x = 5$ or 1

and $y = 1$ or 5.