such that $n = s\sigma = t\tau = \ldots\ldots = b\beta$. Let w be a primitive n^{th} root of unity. Then w^m is a primitive fourth root of unity, w^σ a primitive s^{th} root of unity, and so on. Let

$$\left.\begin{array}{l} w^\sigma,\ w^{\sigma\lambda},\ w^{\sigma\lambda^2}, \ldots\ldots,\ w^{\sigma\lambda^{s-2}} \\ w^\tau,\ w^{\tau h},\ w^{\tau h^2}, \ldots\ldots,\ w^{\tau h^{t-2}} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ w^\delta,\ w^{\delta l},\ w^{\delta l^2}, \ldots\ldots,\ w^{\delta l^{d-2}} \\ w^\beta,\ w^{\beta k},\ w^{\beta k^2}, \ldots\ldots,\ w^{\beta k^{b-2}} \end{array}\right\} \tag{101}$$

be cycles containing respectively all the primitive s^{th} roots of unity, all the primitive t^{th} roots of unity, and so on. Let P_1 be a rational function of w, and, for any integral value of z, let P_z be what P_1 becomes by changing w into w^z. We can always take P_1 such that P_m shall have the form of the fundamental element of the root of a pure uni-serial Abelian quartic; that is, P_m may receive the form of R_1 in (49) as determined by the equations (48). For, because P_1 is a rational function of w,

$$P_1 = a + a_1 w + a_2 w^2 + \ldots\ldots + a_{n-1} w^{n-1},$$

the coefficients a, a_1, etc., being rational. Therefore

$$\begin{aligned} P_m &= a + a_1 w^m + a_2 w^{2m} + \text{etc.} \\ &= (a + a_4 + \text{etc.}) + w^m (a_1 + a_5 + \text{etc.}) + w^{2m} (a_2 + \text{etc.}) + w^{3m} (a_3 + \text{etc.}). \end{aligned}$$

This may be written

$$P_m = f + f' w^m + f'' w^{2m} + f''' w^{3m}. \tag{102}$$

All that is required in order that P_1 may be a function of the kind described is that P_m in (102) be of the same character with R_1 in (49). That is, we have to make

$$f = p,\ f' = q,\ f'' = r,\ f''' = s.$$

By means of these four linear equations, the necessary relations between the quantities a, a_1, a_2, etc., can be constituted. Having thus taken P_1 subject to the condition that P_m shall have the form of the fundamental element of the root of a pure uni-serial Abelian quartic, put

$$\left.\begin{array}{l} \phi_\sigma = P_\sigma^{\lambda^{s-2}} P_{\sigma\lambda}^{\lambda^{s-3}} P_{\sigma\lambda^2}^{\lambda^{s-4}} \ldots\ldots P_{\sigma\lambda^{s-2}} \\ \psi_\tau = P_\tau^{h^{t-2}} P_{\tau h}^{h^{t-3}} P_{\tau h^2}^{h^{t-4}} \ldots\ldots P_{\tau h^{t-2}} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ X_\delta = P_\delta^{l^{d-2}} P_{\delta l}^{l^{d-3}} P_{\delta l^2}^{l^{d-4}} \ldots\ldots P_{\delta l^{d-2}} \\ F_\beta = P_\beta^{k^{b-2}} P_{\beta k}^{k^{b-3}} P_{\beta k^2}^{k^{b-4}} \ldots\ldots P_{\beta k^{b-2}} \end{array}\right\} \tag{103}$$