Therefore, from (47), by the elimination of A',

$$4c^2z = \frac{\left[\frac{1}{4}g^2p_5 + k\{2(k^2-c^2z)-g^3\}\right]^2}{(k^2-c^2z)^2+g^6-2g^3(k^2+c^2z)}. \qquad (49)$$

But, by (45), c^2z is known in terms of the coefficients of the given quintic. Therefore the equation (49) gives a relation necessarily subsisting between the coefficients p_2, p_3, p_4 and p_5 of the solvable quintic in which $u_1u_4 = u_2u_3$, in order that u_1u_4 may be equal to u_2u_3. To find now the root of the quintic, c^2z is known by (45), and B is known by (46). To find $B'\surd z$, making use of the values of P and aQ and $(A'\surd z)^2$ obtained above, we have, from the second of equations (34),

$$g^3(B'\surd z) = c\surd z\{2(k^2-c^2z)+g^3\} + 2k(A'\surd z). \qquad (50)$$

Now, by the first of equations (34), keeping in view the value of azA' in (35),

$$\begin{aligned} g^3B &= g^3\{2k(k^2-c^2z)-g^3k\} + 2azA'\left(\frac{g^2c}{a}\right) \\ &= g^3\{2k(k^2-c^2z)-g^3k\} + 2g^2czA'. \end{aligned}$$

As B and c^2z are known, this equation determines the sign of czA' or $(c\surd z)(A'\surd z)$, and therefore determines the signs with which $c\surd z$ and $A'\surd z$ are to be taken relatively to one another. Hence $z(A')^2$ being given by (48), $B'\surd z$ is given by (50). Therefore $B + B'\surd z$ is known. And $u_1u_4 = g$. Therefore, because

$$u_1^5 = B + B'\surd z + \surd\{(B+B'\surd z) - g^5\},$$

the root of the quintic is known.

§29. *Fourth Example.*—As an illustrative example, let

$$x^5 - \frac{22}{5}x^3 - \frac{11}{25}x^2 + \frac{11\times 42}{125}x + \frac{11\times 89}{3125} = 0.$$

Finding the value of c^2z from (45), and substituting in (49), we find that (49) is satisfied. Then, as in the preceding section,

$$p_5 = -4B \;\therefore\; B = -\frac{11\times 89}{4\times 5^5}.$$

The value of c^2z is $\frac{5}{16}\left(\frac{11}{25}\right)^2$. Therefore

$$k^2 - c^2z = \left(\frac{11}{25}\right)^2\left(\frac{1}{400}-\frac{5}{16}\right) = -\frac{31}{100}\left(\frac{11}{25}\right)^2,$$

and

$$k^2 + c^2z = \left(\frac{11}{25}\right)^2\left(\frac{1}{400}+\frac{5}{16}\right) = \frac{63}{200}\left(\frac{11}{25}\right)^2.$$