

9. How is the proportionality of four numbers expressed in Algebra? State and prove the propositions quoted by the words "Alternando," "Componendo," "Dividendo" and "Ex aequali."

There are four pairs of quantities, the ratio of the second pair is equal to that compounded of the ratios of the first and third pairs, and the ratio of the third pair to that compounded of the ratios of the second and fourth. Shew that the first pair are inversely as the last.

10. When is one quantity said to vary as another, directly or inversely?

If a varies as d^2 , b^3 as d^4 and c^3 inversely as d , shew that the product abc varies as if each of them varied directly as d .

11. Investigate the expression for the sum of n terms of an arithmetic progression whose first term is a and common difference d .

Shew that n consecutive odd numbers beginning with $2m + 1$ exceeds the sum of the first n odd numbers by $2mn$.

12. Find the arithmetic, geometric and harmonic means between two numbers a and b .

If the numbers are nearly equal, shew that the differences between the means are very small compared with the difference between the numbers themselves.

13. Find the sum of n terms of a descending geometric series, and the limit of the sum when it is continued *ad infinitum*.

If the number of terms be odd, shew that the middle term is the n th root of the product of all the terms.

14. Investigate the expressions for the number of permutations and the number of combinations of n things taken r together.

There are four sets of counters distinguished by different colors; each set contains six counters, marked with the numerals from one to six. One counter being taken from each set find the number of different collections which can be formed—the sum of the numbers in each collection being 9.

SOLUTION TO PROBLEMS FROM CORRESPONDENTS.

1. The diagonals AC, BD, of a parallelogram intersect in O, and P is a point within the triangle AOB. Prove that the difference between the triangles APB, CPD, is equal to the sum of the triangles APC, BPD.

Join PO.

$$\therefore DPC - DOC = DPO + CPO, \text{ and } AOB - APB = BPO + APO,$$

$$\therefore \text{by addition, since } AOB = DOC, \text{ we get } DPC - APB = DPO + BPO + CPO + APO = DPB + APC.$$

2. If from a point without a parallelogram there be drawn two straight lines to the extremities of the two opposite sides between which, when produced, the point does not lie, the difference of the triangles thus formed is equal to half the parallelogram.

Let ABCD be the parallelogram, and let O be the point, and let O lie on the side of AB remote from DC; through O draw FOE parallel to AB, produce DA, CB, to meet FE in F and E, and join OA, OB, OC, OD.

Then since the triangle ODC is half the parallelogram FDCE, and OAB is half of FAGE, therefore the difference between ODC and OAB is equal to half the difference between FDCE and FAGE, that is, to half the parallelogram ABCD.

3. Two cisterns of equal size are emptied by two taps in 4 and 5 hours respectively. If the taps are opened when both cisterns are full in what time will one cistern contain twice as much water as the other?

(1) Let x = required time in hours,

Then in x hours $\frac{x}{4}$ of the first will be emptied

and $\frac{x}{5}$ of the second, $\therefore$ the first will still

contain $\left(1 - \frac{x}{4}\right)$ of its full capacity, and

the second $\left(1 - \frac{x}{5}\right)$ and since $\left(1 - \frac{x}{4}\right)$