Mathematical Department.

We find it necessary to devote all our space this month to dispose of the correspondence that has accumulated on our hands. It is a great encouragement to have such an army of friends, and we regret that our limits compel us to condense, curtail, or pass over much of what they send us. Nevertheless, we hope they will continue to write, and thus keep us fully acquainted with their particular wants. We aim at making this Department above all things *useful* to our subscribers, and our warmest sympathies go out to those who have not the advantage of teachers and professors to assist them. To our correspondents, one and all, we return hearty thanks and bid them cordial greeting.

In order that our shortcomings may not be placed on his shoulders, we beg to state that Mr. Baker's connection with this Department ceased with the February number of 1881. All correspondence should be addressed to THE MATHEMATICAL DEPARTMENT, and should be written only on one side of the paper, and should be kept separate from other communications.

L. J. CORNWALL, Beamsville, sends solution of No. 7, part 2, page 126, as already published, page 152. He wishes for the success of the JOURNAL, and would be thankful to receive a solution of the following:—Given base, vertical angle, and length of bisector of vertical angle, to construct the triangle. He will find the solution in Thompson's Euclid, Appendix Bk. III., Prop. X.

A. HAY, Barrie, also sent solution of No. 7, part 2, p. 126.

JANE SHUE, Sandwich East, sends a neat solution of No. 1, p. 150.

JOHN MOSER, South Tay, N.B., sends solutions to four problems on page 150.

MRS. G. C. WARBURTON, Toronto, sent full solutions to problems on page 78. Her results are: 36 men; 100.42; $x=\pm\frac{2}{?}$ or $\pm\frac{7}{12}$, $x=\sqrt{(-\frac{a^2}{3}}$; $x=\frac{5}{3}$, $y=\frac{5}{3}$, $z=\frac{5}{3}$.

JOHN ANDERSON, Dixie, Ont., finds 1154·348 gals., U. S. wine measure, to be the amount of oil in cylinder mentioned on p. 152. We regret that the geometrical figure and the length of the solution preclude publication in this issue. Mr. Anderson makes 180 days and not 100 days the answer to No. 3, p. 150. We think Mr. Moser is correct with 100 days. Mr. A. also gets a smaller result than that we gave for No. 2, p. 150. Perhaps the discrepancy arises from the use of five-figure logarithms instead of higher tables with eleven. Will our friends investigate? Mr. A.'s result is:—

$39841243491743119266055045872293577981,137614·67.

We took our answer from a mathematical journal.

ALEX. KERR, Wiarton, asks the price of a mortgage of $650 due in 5 years, 3 months of which have expired, interest @ 7 per cent., payable yearly, so that the purchaser may make 9 per cent. on his investment. We assume that compound interest is meant, as is usual in such cases.

D. R. BOYLE, West Arichat, C.B., sent a solution of oil question on page 152:

Area of end of cylinder = 19·63495
" part unfilled = 6·42428

Area of part filled = 13·21067

Giving 823·3 gals. of 277·274 cubic inches each, which is the same as Mr. Anderson's result.

We have to stop here to leave room for a veteran mathematician, whose appearance in our columns will be hailed with delight. The following explains itself:—

CAMBRIDGE, August 16th, 1883.

Mathematical Editor, CANADA SCHOOL JOURNAL:—

DEAR SIR,—I have just laid my hand on a paper on "Converse Propositions," written by the late T. S. Davies, Esq., who held the second Mathematical chair at the Royal Military Academy at Woolwich. I always regarded him as the most eminent geometer of his time. It has occurred to me you might be able to make some use of this paper in your SCHOOL JOURNAL, which I observe takes notice of matters connected with elementary mathematics.

Yours most truly,

R. POTTS.

CONVERSE PROPOSITIONS.

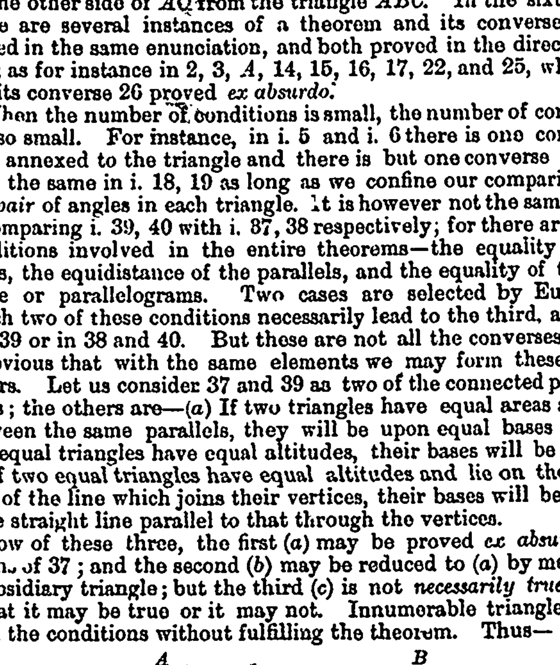
There are many careless writers who deem it sufficient to prove one proposition and to assume *thence* the truth of its converse. Thus, if it had been proved that the angles at the base of an isosceles triangle are equal, they would assume without further proof, that if the angles at the base of a triangle be equal, the sides opposite to them were also equal. In the same way, but with more plausibility, they consider that i. 19 is the converse of i. 18 and needs no proof whatever. In a certain sense i. 17 is the converse of i. *ax.* 12; and hence one or the other of these would by such writers be assumed as true, according to which was first admitted; and as i. 17 is admitted to be rigidly proved, they ought, for the sake of consistency, to make *ax.* 12 a corollary of i. 17 instead of keeping it in its present place.

Although Euclid does not always discuss the converse propositions, he certainly never assumes them as corollaries in the manner described. He does not enunciate them at all, except they be such as he requires in the ulterior part of his writings; and then he always gives them the form of distinct propositions with the requisite constructions and demonstrations, as the cases may demand. In modern research we require a much greater number of theorems and elementary problems than he did; and amongst these wants are the converses of some of his direct propositions, and indeed of much more complex ones.

With respect to problems, however, the converse is often of a totally different nature from the direct one; indeed, so much so that we can sometimes gain but little assistance from the one construction in devising the other. In theorems, on the other hand, the method *ex absurdo* will in general effect the purpose, if no direct method of proof should present itself to the mind. Still, wherever the direct proof can be obtained it is to be preferred; and the indirect only employed in cases where the other fails to suggest itself to the mind. In the propositions quoted (i. 5 & 6, i. 18 & 19) and several others in the first and third books, the *ex absurdo* method is employed; but in i. 48 it is evaded by drawing the triangle ADC on the other side of AC from the triangle ABC. In the sixth book there are several instances of a theorem and its converse being joined in the same enunciation, and both proved in the direct manner; as for instance in 2, 3, A, 14, 15, 16, 17, 22, and 25, whilst 24 has its converse 26 proved *ex absurdo*.

When the number of conditions is small, the number of converses is also small. For instance, in i. 5 and i. 6 there is one condition only annexed to the triangle and there is but one converse of i. 5. It is the same in i. 18, 19 as long as we confine our comparison to *one pair* of angles in each triangle. It is however not the same again in comparing i. 39, 40 with i. 37, 38 respectively; for there are three conditions involved in the entire theorems—the equality of the bases, the equidistance of the parallels, and the equality of the triangle or parallelograms. Two cases are selected by Euclid in which two of these conditions necessarily lead to the third, as in 37 and 39 or in 38 and 40. But these are not all the converses; as it is obvious that with the same elements we may form these three others. Let us consider 37 and 39 as two of the connected propositions; the others are—(*a*) If two triangles have equal areas and be between the same parallels, they will be upon equal bases; (*b*) If two equal triangles have equal altitudes, their bases will be equal; (*c*) If two equal triangles have equal altitudes and lie on the same side of the line which joins their vertices, their bases will be in the same straight line parallel to that through the vertices.

Now of these three, the first (*a*) may be proved *ex absurdo* by means of 37; and the second (*b*) may be reduced to (*a*) by means of a subsidiary triangle; but the third (*c*) is not *necessarily true*—that is that it may be true or it may not. Innumerable triangles may fulfil the conditions without fulfilling the theorem. Thus—



Let AB be the vertices, AP, BQ the equal altitudes of the triangles ACD and BEF; describe semicircles about A, B with these equal altitudes as radii; draw tangents CD, EF at P, Q, and make them equal to one another (but not necessarily having $CP=QF$ and hence $PD=QE$, nor even so that C, D shall be on different sides of P,