$= 2c \sin B \cos A + 2a \sin C \cos B + 2b \sin A \cos C$
$= \sin A\,(b \cos C + c \cos A) + \sin B\,(a \cos C + c\, A) + \sin C\,(b \cos A + a \cos B)$
$= a \,.\, \sin A + b \sin B + c \sin C.$

9. (*a*) In any triangle $a^2 = b^2 + c^2 - 2bc \cos A$. Show from this that if c has two real positive values a is less than b, and the triangle is ambiguous.

(*b*) If in the ambiguous case the ratio of the two values of the indeterminate side be $\sqrt{3+2}$, and the given angle be 45°, show that the angle between the two positions of the opposite side is 60°.

9. (*a*) $a^2 = b^2 + c^2 - 2bc \cos A$. Solve for c.

$$c^2 - 2c \,.\, b \cos A + \left(\frac{b \cos A}{2}\right)^2 = a^2 - b^2 + b^2 \cos^2 A,$$

$$\therefore\ c = b \cos A \mp \sqrt{a^2 - b^2 + b^2 \cos^2 A}.$$

If both values of c are positive;

$$b^2 \cos^2 A > a^2 - b^2 + b^2 \cos^2 A.$$
$$\text{or } b^2 > a^2 \,.\quad \text{or } a < b.$$

One side c having two positive values, and a being $< b$, the triangle is ambiguous.

(*b*) (1) $\dfrac{b_1 + b_2}{2} = AD = \dfrac{c}{\sqrt{2}}$.

$$\therefore\ b_1 + b_2 = b\sqrt{2}\,.$$

(2) $b_1 + b_2 :: \sqrt{3}+2 : 1.$

From (2) $b_1 = b_2\,(\sqrt{3}+2)$ substitute in (1). $b_2\,(\sqrt{3}+2+1) = c\sqrt{2}$,

$$\text{or } b_2 = \frac{c\sqrt{2}}{\sqrt{3}+3}.$$

$$\therefore\ b_1 = \frac{c\sqrt{2}(\sqrt{3}+2)}{\sqrt{3}+3}.$$

$$\therefore\ \tan a = \frac{\dfrac{b_1 - b_2}{2}}{\dfrac{c}{\sqrt{2}}} = \frac{\dfrac{c\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}+1}{\sqrt{3}+3}}{\dfrac{c}{\sqrt{2}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}+3} = \frac{1}{\sqrt{3}}.$$

$\therefore\ a = 30°$ and angle $CBC' = 60°$.

3

10. (*a*) Given a, b and C, write formula for finding A, B and c.

(*b*) The radii of two wheels, in the same plane are R and r, and a belt goes around them and crosses between them at an angle 2θ. Find the length of the belt, and show that the length is constant while the sum of the radii is constant.

10 (*a*) Book work.

(*b*) The length of the st part $= 2r \cot \theta + 2R \cot \theta = 2 \cot \theta\,(r+R)$.

Curved parts: Angle $P = \pi + 2\theta$ are subtended by this angle $= (\pi + 2\theta)(r+R)$.
Total length $= (r+R)\,(2 \cot \theta + \pi + 2\theta)$ which is constant if $r+R$ is constant.

11. (*a*) ABC is an equilateral △, and E, on BC is a vertex of the inscribed square whose side lies along AC. Show that $\tan EAC = \frac{1}{2}(3-\sqrt{3})$.

(*b*) The altitude of a certain rock is $a°$, and after walking b feet towards the rock up a slope of $\beta°$ to the horizon the altitude of the rock is then $\gamma°$. Find the vertical height of the rock above the first position.

11. (*a*) $\operatorname{Tan} EAC = \dfrac{ED}{DA}$.

$$DC = x \,.\, \cot c = x \cot 60° = \frac{x}{\sqrt{3}}.$$

$$\operatorname{Tan} EAC = \frac{x}{x + \dfrac{x}{\sqrt{3}}}$$

$$= \frac{x}{x}\left(\frac{\sqrt{3}}{\sqrt{3}+1}\right) = \frac{\sqrt{3}}{\sqrt{3}+1}.$$

$$\therefore\ \operatorname{Tan} EAC = \frac{3-\sqrt{3}}{3-1} = \tfrac{1}{2}(3-\sqrt{3}).$$

(*b*) Let $A + B$ be the two positions of the observer upon the inclined plane of $< B$, and let C be the top of the tower. Then the angle $ACB = (\gamma - a)$ and we have:

$$\frac{AC}{\sin \gamma} = \frac{b}{\sin(\gamma - a)} \text{ or } AC = \frac{b \sin \gamma}{\sin(\gamma - a)}.$$

and the height of the tower $= AC \sin(a+B)$

$$= \frac{b \sin \gamma \,.\, \sin(a+\beta)}{\sin(\gamma - a)}.$$