Work done up to any instant plus kinetic energy at that instant is equal to the initia kinetic energy plus work available.

$\int (ds - dy)p$ = work done in compressing the fibres at the point of contact of the tup, s being distance passed through by the tup, y the distance the centre of gravity of the beam passes through, and p the pressure between the tup and the beam. The difference between y and s is usually less than ½ of 1 per cent. of the total deflection, so that the work lost in denting the beam is negligibly small and will be neglected in the subsequent discussion.

$\int F dy$ = work done in deflecting the beam. The force F is practically identical with the pressure p, and would be exactly the same as p for weightless beams. F is in the determination of this force that the main problem of stresses in beams subject to impact lies. F is equivalent to the centre load in a static bending test, which, when plotted against deflections, gives a curve from which the modulus of elasticity, the modulus of rupture, and the energy of rupture may be computed.

$\int \frac{\delta W_b}{2g} \mu^2$ = kinetic energy of the beam, where W_b is the weight of the beam and μ the velocity of any element $\delta W_b{}^1$.

$\frac{1}{2} \frac{W_t}{g} v^2$ = kinetic energy of the tup at any instant under consideration after initial contact, W_t being the weight of the tup.

E_e = energy lost in vibrations. In all impact some energy is lost in vibrations, but this can be largely reduced by making the ratio of the weight of the machine frame to that of the tup relatively large. It will be omitted on this basis.

$\int W_t ds$ = work done by gravity on the tup after initial contact with the beam.

$\int \delta W_b z$ = work done by gravity on the beam after initial contact, z being the deflection of the element δW_b.

$\frac{1}{2} \frac{W_t}{g} v_t{}^2$ = kinetic energy of the tup at the instant of contact, v_t being the velocity of the tup at that instant.

While equation (2) is general and applies at any instant during the motion, the energy that exists as vibrations will be relatively large for the first part of the deflection during the time required to overcome the inertia of the beam. Consequently the subsequent discussion will concern the motion after the vibrationless condition has been more nearly reached.

Referring again to the integral $\int F dy$, it will now be observed that the work of deflection is equal to the change in kinetic energy of the tup plus the work done by gravity on the tup and beam during deflection, minus the energy lost in imparting velocity to the beam and denting it at the point of contact.

Bearing in mind that v is the velocity at the centre after the inertia of the beam has been entirely overcome, and u is the corresponding velocity of any element δW_b, away from the centre, it is a simple matter to express u in terms of v, since the elastic curve is assumed to be known. Let l be the length of the beam, y' and z', y'' and z'' be the corresponding deflections at the centre and at any section distant x from the end, respectively, and let P' and P'' be the loads at the centre corresponding to y' and y''. Since the load at the centre is directly pro-

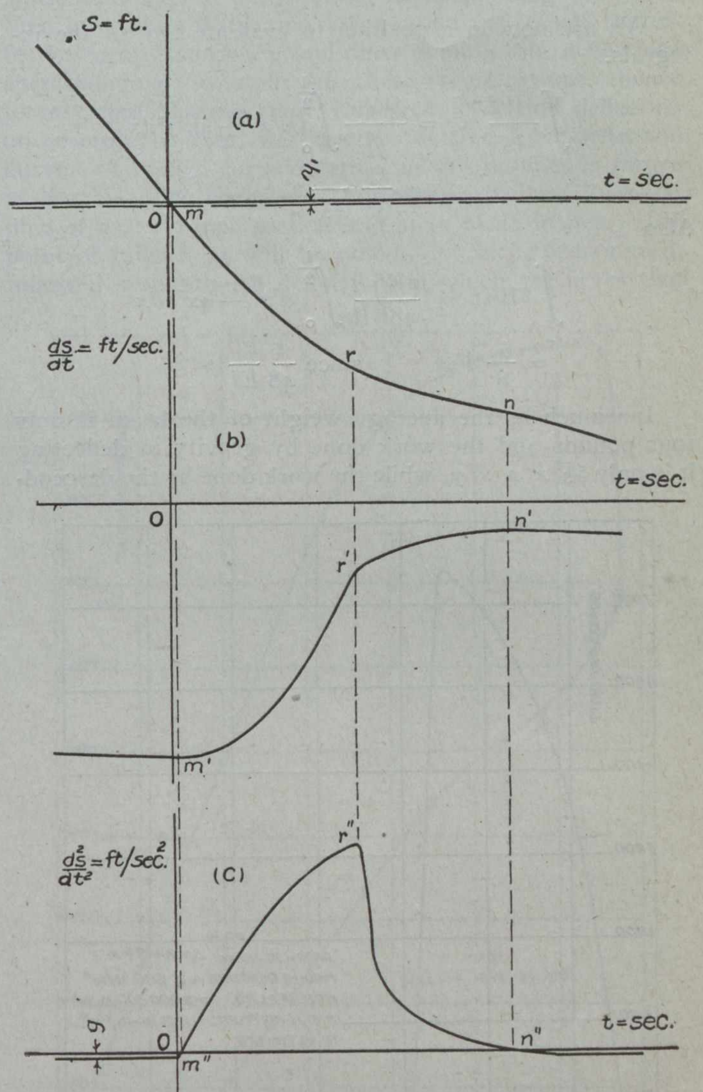

Fig. 1.—A Typical Time-Deflection Curve from Impact Test on a Timber Beam, and Its First and Second Differential Curves

portional to the deflection at the centre, or at any other point with the elastic limit

$$\frac{y'}{y''} = \frac{P'}{P''} = \frac{z'}{z''}$$

$$\frac{y'' - y'}{y'} = \frac{z'' - z'}{z'}$$

But $y'' - y' = dy$, and $z'' - z' = dz$, hence

$$\frac{dy}{y'} = \frac{dz}{z'}$$

$$\frac{dy}{dt} \; \frac{1}{y'} = \frac{dz}{dt} \; \frac{1}{z'}$$

$$\text{or } \frac{v}{y'} = \frac{u}{z'} \qquad \text{and } u = v\frac{z'}{y'}$$

The deflection at any section distant x from the left end of the beam is

$$z' = \frac{P'}{48EI}(3l^2x - 4x^3)$$