

A clearer understanding of the basis of geometry prepares us to appreciate the advance in geometrical knowledge due to Greek intellect. "The first name," says Allman, "which meets us in the history of Greek mathematics is that of Thales of Miletus (640-536 B.C.). . . . Thales himself was engaged in trade, is said to have resided in Egypt, and, on his return to Miletus in his old age, to have brought with him from that country the knowledge of geometry and astronomy. To the knowledge thus introduced he added the capital creation of the geometry of lines, which was essentially abstract in its character. The only geometry known to the Egyptian priests was that of surfaces, together with a sketch of that of solids . . . obtained empirically; Thales, on the other hand, introduced *abstract* geometry, the object of which is to establish precise *relations* between the different parts of a figure, so that some of them could be found by means of others in a manner strictly rigorous. This was a phenomenon quite new in the world, and due, in fact, to the abstract spirit of the Greeks."

"In connection with the new impulse given to geometry, there arose with Thales, moreover, scientific astronomy, also an abstract science, and undoubtedly a Greek creation. The astronomy of the Greeks differs from that of the Orientals in this respect—that the astronomy of the latter, which is altogether concrete and empirical, consisted merely in determining the duration of some periods, or in indicating, by means of a mechanical process, the motion of the sun and planets; whilst the astronomy of the Greeks aimed at the discovery of the geometric laws of the motions of the heavenly bodies." Thales "measured the Pyramids, making an observation on our shadows when they are of the same length as ourselves, and applying it to the Pyramids. . . . Thales meas-

ured the distance of vessels from the shore by a geometric process." Note these applications to the concrete. Again, we are told by the historian Eudæmus that he attempted "some things in a more abstract manner, and some in a more intuitional or sensible manner." Thus it is clear that he would continue to employ empirical measurements to obtain approximate results, which, by the creation of definitions and the use of axioms, he would gradually replace by strictly scientific theorems. Allman attributes to Thales the discovery of the two theorems—(a) The sum of the three angles of a triangle is equal to two right angles; (b) The sides of equiangular triangles are proportional. (Hence the basis of the theory of *similar* figures.) Thus, from a philosophic point of view, says Allman, "we see in these two theorems of Thales the first type of a *natural law*—i.e., the expression of a fixed dependence between different quantities, or, in another form, the disentanglement of constancy in the midst of variety—has decisively risen"; whilst, from a practical point of view, "Thales furnished the first example of an application of theoretical geometry to practice, and laid the foundation of an important branch of the same—the measurement of heights and distances." After Thales comes the contribution of the Pythagorean school. "Pythagoras changed geometry into the form of a liberal science, regarding its principles in a purely abstract manner, and investigated his theorems from the immaterial and intellectual point of view." He was the first person who introduced weights and measures among the Greeks. The geometry of areas plays an important part in the work of this school (e.g., Euclid I. 47), thus exhibiting the mode of evolution from its Egyptian empirical source. Again, "the Pythagoreans first severed geometry from the needs of practical life, and treated it as a liberal science, giv-