the arc de, and *ef*, through which, if arcs be described in the same manner, from the points B and D′ as centers, and intersecting the arcs ka, and *i*b, that the intersections a, b, and c, or the curve line abc ... n, shall be on the chord an.

For let the point d move on the arc d*e* towards the point *e*, and the point *e* move towards *f*, in like manner the point of intersection a will move towards a′, and the intersection b will move towards b′—and let the point d arrive at the point *e*, in the same time the point *e* arrives at the point *f*—the point a, must equally arrive at the point a′ and the arcs da and *e*b, coincide with the arcs *e*a′, and *f*b′; but the points b and c have changed sides to the chord an, for the curve line abc ... n, will now coincide with the curve line a′b′c′ ... n—consequently there must be points of variation passed on the arcs d*e*, and *ef*, through which if intersections be described from the centers D′ and B, that the curve line abc ... n will become a straight line, or that the intersections a, b, c, will be on a straight line with the point n, and coincide with the chord an.

LEMMA 8.

From the center C of the semicircle ABDE, describe through the point of intersection *s*, the arc *ss*′, meeting the arc 1T, in the point *s*′, and from the same center describe through the intersection r, the arc rr′, meeting the arc 2S, in the point r′—and from K as a center with the distances Ks′ and Kr′, describe the arcs s′s″ and r′r″.—The point s″, shall be the greatest variation of the point t′ towards the point s, on the intercepted arc 1″K of the arc BK,—and the point r″, shall be the greatest variation of the point s towards the point r, and the arcs ts″ and sr″, shall be proportional to the arcs 12, and 23.* FIG. 7.

For by construction, the arc 1 2 3 ... 4′3′2′ on the arc BD, is the least intercepted arc by the series of arcs 1K, and the arc 1′K is the greatest of all the intercepted arcs of the radius AC, which can be described through B, between the arcs BD and BC, meeting the arc CD,—therefore the arcs t′s″, and sr″, are the greatest variations of the points t and r, because the greatest intercepted arc is t′K.

From the point A as a center, through the point of intersection t of the arcs 1″T′ and BK′, describe the arc tu′ meeting the arc 2″S′ in the point u′ and from the same center through the point of intersection u, describe the arc uv′—and from K′ as a center describe through the points u′ and V′—the arcs u′u″, and v′v″.—The arcs tu″ and uv″, must be the greatest variations of the point t towards u, and of u towards v, because the greatest intercepted arc is tK′

Now let the arc BK, move on the point B as a center, until it coincides with the arc BD, it is evident that the variable curvilinear triangles ts′s, and sr′r, must vanish or at the same time or become nothing; because the arcs s′s, and r′r, must always remain concentric to the arc BD,—and therefore the variations t′s″ and sr″, equally vanish, when the arcs s′s and r′r coincide with the arcs 12 and 23, on which arcs, the variations come to be nothing. In the same manner when the arc BK′ comes to coincide with the arc BC, the variations tu″ and uv″ vanish; for the arcs tu′ and uv′, are each concentric to the arc BC; consequently the ultimate ratios of the variations on the arcs BK and BK′ are equal; and the variation ts″ is to the variation sr″, as the variation tu″, is to the variation uv″; also the ultimate ratios, of ts and sr, are equal, and therefore t′s is to sr as the arc 12 is to the arc 23; but the ultimate ratios of t′s″ and sr″, and the ultimate ratios of t′s and sr are equal; for when the arcs ts and sr coincide with the arcs 12 and 23, the variations t′s″ and sr″ vanish at the same time; therefore the variation t′s″ is to the variation sr″, as the arc 12 is to the arc 23.

LEMMA 9

Bisect the arc CD, in the point D′, and through the points of intersection (Lem. 1,) B, d, e, f, ... h, i, k, D′, draw the curve line Bdef ... hikD′, and let the lines Btuv .. q′o′K′, and Bt′sr ... qoK, be each described similar and equal to the line Bdef ... hikD′—and bisect the distance BD′, by the straight line AH, intersecting the curve line in the point G.—Next from the point D′, with the distance D′*e*′ and D′*f*′, describe the arcs *ee*″ and *ff*″; The point *e*″ shall be the variation of the point d towards the point *e*, and the point *f*″, shall be the variation of the point *e* towards the point *f*, on the intercepted arc dk, which is common to the series of arcs 1K and 1″K—t′s″ shall be to sr″, as tu″ is to uv″, and as de″ is to ef″. FIG. 7.

* This is, supposing that the arcs BD, BK′, BK and BC, are similar and equal to each other, or of the same radius AC.