

7. Given $x = 3$, $y = 4$, $z = -5$, find the values of

$$(x + y + z)^3 - 3(x + y + z)(xy + yz + zx);$$

$$x^2(y + z) + y^2(z + x) + z^2(x + y) + 2xyz;$$

$$x^2(y - z) + y^2(z - x) + z^2(x - y);$$

$$(5x - 4z)^2 + 9(4x - z)^2 - (13x - 5z)^2;$$

$$(3x + 4y + 5z)^2 + (4x + 3y + 12z)^2 - (5x + 5y + 13z)^2.$$

8. If $s = a + b + c$, find the values of

$$(2s - a)^2 + (2s - b)^2 - (2s + c)^2$$

when

(i.) $a = 3$, $b = 4$, $c = 5$;

(ii.) $a = 21$, $b = 20$, $c = 29$;

(iii.) $a = 119$, $b = 120$, $c = 169$;

(iv.) $a = 3$, $b = -4$, $c = 5$;

(v.) $a = 5$, $b = 12$, $c = -13$.

9. If $a = 1$, $b = 3$, $c = 5$, $d = 7$, $e = 9$, $f = 11$, show that

$$a + b + c + d + e + f = \left(\frac{a+f}{2}\right)^2;$$

$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{cd} + \frac{1}{de} + \frac{1}{ef} = \frac{1}{2}\left(\frac{1}{a} - \frac{1}{f}\right);$$

$$\frac{1}{abc} + \frac{1}{bcd} + \frac{1}{cde} + \frac{1}{def} = \frac{1}{4}\left(\frac{1}{ab} - \frac{1}{ef}\right);$$

$$\frac{1}{abcd} + \frac{1}{bcde} + \frac{1}{cdef} = \frac{1}{6}\left(\frac{1}{abc} - \frac{1}{def}\right);$$

$$a^2 + b^2 + c^2 - ab - bc - ca = b^2 + c^2 + d^2 - bc - cd - db$$

$$= c^2 + d^2 + e^2 - cd - de - ec$$

$$= d^2 + e^2 + f^2 - de - ef - fd.$$