In like manner, by putting c for c in (109), and taking the result in connection with the first of equations (127),

$$(R_{cv}R_c^{-v})^{\frac{1}{n}} = w^r (A_{cv}A_c^{-v})Q(F_{c\beta}^{-v\beta}\ldots.)^{\frac{1}{n}}\{(P_{cvm}^m P_{cm}^{-vm})(\phi_{cv\sigma}^\sigma \phi_{c\sigma}^{-v\sigma})\ldots.\}^{\frac{1}{n}}. \tag{130}$$

From (129) compared with the first of equations (128), and from (130) compared with the second of equations (128),

$$\left.\begin{aligned} k_1 &= w^a (A_v A_1^{-v}) Q (F_\beta^{-v\beta}\ldots.)^{\frac{1}{n}}\{(P_{vm}^m P_m^{-vm})(\phi_{v\sigma}^\sigma \phi_\sigma^{-v\sigma})\ldots.\}^{\frac{1}{n}} \\ \text{and}\qquad k_c &= w^r (A_{cv}A_c^{-v}) Q (F_{c\beta}^{-v\beta}\ldots.)^{\frac{1}{n}}\{(P_{cvm}^m P_{cm}^{-vm})(\phi_{cv\sigma}^\sigma \phi_{c\sigma}^{-v\sigma})\ldots.\}^{\frac{1}{n}} \end{aligned}\right\} \tag{131}$$

Exactly as in § 44, it can be shown that

$$(\phi_{cv\sigma}^\sigma \phi_{c\sigma}^{-v\sigma})^{\frac{1}{n}} = q_c, \tag{132}$$

q_c being a rational function of the primitive n^{th} root of unity w^c. Also, it has been proved that P_m is of the form of the fundamental element of the root of a pure uni-serial Abelian quartic. Therefore, by (3), $(P_{cvm}P_{cm}^{-v})^{\frac{1}{4}}$ is a rational function of the primitive fourth root of unity w^m. Therefore, because $n = 4m$, $(P_{cvm}^m P_{cm}^{-vm})^{\frac{1}{n}}$ is a rational function of the primitive n^{th} root of unity w^c. Put

$$(P_{cvm}^m P_{cm}^{-vm})^{\frac{1}{n}} = q_c'. \tag{133}$$

Again, exactly as in § 44,

$$F_{cv}^{-\frac{v\beta}{n}} = q_c'', \tag{134}$$

q_c'' being a rational function of w^c. By (132), (133), (134), and other corresponding equations, the second of equations (131) becomes

$$k_c = w^r (A_{cv}A_c^{-v})\, Q\, (q_c q_c' q_c'' \ldots.). \tag{135}$$

In like manner, from the first of equations (131),

$$k_1 = w^a (A_v A_1^{-v})\, Q\, (q_1 q_1' q_1'' \ldots.),$$

q_1, q_1', etc., being what q_c, q_c', etc., become in passing from w^c to w. It may be noted that this assumes that we are entitled to change equation (133) into

$$(P_{vm}^m P_m^{-vm})^{\frac{1}{n}} = q_1'.$$

The warrant for this lies in the fact that the roots $P_m^{\frac{m}{n}}$, $P_{2m}^{\frac{m}{n}}$, $P_{3m}^{\frac{m}{n}}$, or $P_m^{\frac{1}{4}}$, $P_{2m}^{\frac{1}{4}}$, $P_{3m}^{\frac{1}{4}}$, were taken with the values they have in the root

$$P_0^{\frac{1}{4}} + P_m^{\frac{1}{4}} + P_{2m}^{\frac{1}{4}} + P_{3m}^{\frac{1}{4}}$$

of a pure uni-serial Abelian quartic. This being so, the equation

$$(P_{vm}^m P_m^{-vm})^{\frac{1}{n}} = q_1'$$

corresponds to equation (3), while (133) corresponds to (5), and, by § 5, equations