direction with the velocity V. The absolute velocity of the water is therefore

$$v - V = \sqrt{V^2 + 2gh} - V. \qquad (2)$$

Now the useful work done per second by each pound of water must equal the work due to the height h, diminished by the work remaining in the water after leaving the machine. Hence,

$$\text{useful work} = h - \frac{(v - V)^2}{2g}$$

$$= \frac{\left(\sqrt{V^2 + 2gh} - V\right) V}{g}, \text{ from (2)}, \qquad (3)$$

$$= \frac{(v - V)\, V}{g}. \qquad (4)$$

The whole work expended by the water fall is h foot-pounds per second; consequently, to find the efficiency of the machine, we divide (3) by h (Anal. Mechs., Art. 216), and get

$$\text{efficiency} = \frac{\left(\sqrt{V^2 + 2gh} - V\right) V}{gh} \qquad (5)$$

$$= 1 - \frac{gh}{2V^2} + \text{etc.} \qquad (6)$$

(by the Binomial Theorem),

which increases towards the limit 1 as V increases towards infinity. Neglecting friction, therefore, the maximum efficiency is reached when the wheel has an infinitely great velocity of rotation. But this condition is impracticable to realize; and even at practicable but high velocities of rotation, the prejudicial resistances, arising from the friction of the water and the friction upon the axis, would considerably reduce the efficiency. Experiment seems to show that the best efficiency of these machines is reached when the velocity is that due to the head, so that $V^2 = 2gh$.