12. Find the ratio of a to b in order that the equations
$ax^2+bx+a=0$, and $x^3-2x^2+2x-1=0$, may have either one or two roots in common.

$$ax^2+bx+a=0, \therefore x^2+\frac{b}{a}x+1=0, \quad (1)$$

and $x^3-2x^2+2x-1=0$,

$$\therefore (x-1)(x^2-x+1)=0. \quad (2)$$

1st. If these two equations have one rational root in common, it is $x=1$; by substituting this value of x in equation (1) we obtain $\frac{b}{a}=-2$.

2nd. If they have a rational quadratic factor it must be (x^2-x+1), and in order that $x^2+\frac{b}{a}x+1$ may contain this factor, $\frac{b}{a}$ must equal -1.

13. Solve

(1) $\frac{1}{x}+\frac{1}{y}=2, \quad \frac{1}{x^3}+\frac{1}{y^3}=14.$

(2) $3x^2+15x-2\sqrt{x^2+5x+1}=2.$

(3) $yz=bc, \quad \frac{x}{a}+\frac{y}{b}=1, \quad \frac{x}{a}+\frac{z}{c}=1.$

$$\frac{1}{x}+\frac{1}{y}=2. \quad (1) \qquad \frac{1}{x^3}+\frac{1}{y^3}=14. \quad (2)$$

(1) Cubing each side of equation (1) we obtain $\frac{1}{x^3}+\frac{1}{y^3}+\frac{3}{xy}\left(\frac{1}{x}+\frac{1}{y}\right)=8$, and by substituting the values of $\frac{1}{x^3}+\frac{1}{y^3}$ and $\frac{1}{x}+\frac{1}{y}$ we obtain $\frac{6}{xy}=-6$ or $xy=-1$. Again, from equation (1) we get $x+y=2xy=-2$, and from the equations $x+y=-2$ and $xy=-1$ we obtain

$$x=\pm\sqrt{2}-1 \text{ and } y=\pm\sqrt{2}-1.$$

(2) By adding 3 to each side of the equation we obtain

$$3(x^2+5x+1)-2\sqrt{x^2+5x+1}=5,$$

that is, $3(x^2+5x+1)-2\sqrt{x^2+5x+1}-5=0$.
or $(3\sqrt{x^2+5x+1}-5)(\sqrt{x^2+5x+1}+1)=0$,

$$\therefore 3\sqrt{x^2+5x+1}=5, \quad (1)$$

$$\text{or } \sqrt{x^2+5x+1}=-1 \quad (2)$$

From equation (1) $x=\frac{1}{3}$ or $-\frac{16}{3}$, and from equation (2) $x=0$ or -5.

(3) $yz=bc$ (1), $\frac{x}{a}+\frac{y}{b}=1$ (2), $\frac{x}{a}+\frac{z}{c}=1$ (3)

From equations (2) and (3) we obtain $\frac{y}{b}$ and $\frac{z}{c}$, and by substituting this value of y in equation (1) $z=\pm c$, and $\therefore y=\pm b$. By substituting value of y in equation (2) we have $x=0$ or $2a$.

PROBLEMS IN ARITHMETIC,

By W. S. Ellis, B.A., Mathematical Master, Cobourg Collegiate Institute.

I. A druggist buys an article at \$2.50 per lb. avoir.; at what price per oz. troy must he sell it so as to gain $\frac{1}{4}$ of cost?
Ans. $21\frac{3}{7}$c.

II. What was the cost per bush. o. wheat that was sold at \$2 per cwt., thus gaining $\frac{1}{8}$ of cost? *Ans.* \$1.06$\frac{2}{3}$.

III. If a piece of land 40 ft. wide and 140 ft. long sell for \$35, what is the price per acre? *Ans.* \$272.25.

IV. A man bought a certain quantity of grain; on $\frac{3}{8}$ of it he gained $\frac{1}{8}$ of cost; on $\frac{1}{8}$ of it he lost $\frac{1}{8}$ of cost; on the remainder he gained \$150. His whole gain being \$560, find the price he paid for the grain.
Ans. \$15,774.

V. $\frac{3}{5}$ of one number is $\frac{5}{8}$ of a second, $\frac{3}{7}$ of the second is $\frac{3}{4}$ of a third. What fraction is the 3rd of the first? *Ans.* $\frac{96}{175}$.

VI. When gold dust is worth \$16.50 per oz., what is it worth in France per gram, taking a gram as $15\frac{2}{5}$ grains Troy, and a franc as 19c.? *Ans.* $2\frac{239}{304}$.

VII. When wheat is worth 58 shillings a quarter, what is it worth in dollars per bush., a quarter being 8 bush. and a sovereign \$4.86$\frac{2}{3}$? *Ans.* \$1.76$\frac{5}{12}$.

VIII. What would be the length in inches of a linear unit, such that the number of units per second traversed by a moving body would be the same as the number of miles per hour? *Ans.* $17\frac{3}{5}$.

IX. A certain quality of silks cost in Paris 15 francs per yd.; freight and insur-