77. To shew $\frac{\sin 8A}{\cos 3A \cos 5A}$

$$=\frac{\sin A+\sin 3A+\sin 5A+\sin 7A+\sin 9A}{\cos A+\cos 3A+\cos 5A+\cos 7A+\cos 9A}$$

$$+\frac{\sin A+\sin 2A+\sin 3A+\sin 4A+\sin 5A}{\cos A+\cos 2A+\cos 3A+\cos 4A+\cos 5A}$$

For

$$\frac{\sin A+\sin 3A+\sin 5A+\sin 7A+\sin 9A}{\cos A+\cos 3A+\cos 5A+\cos 7A+\cos 9A}$$

$$=\frac{(\sin A+\sin 9A)+(\sin 3A+\sin 7A)+\sin 5A}{(\cos A+\cos 9A)+(\cos 3A+\cos 7A)+\cos 5A}$$

$$=\frac{2\sin 5A\cos 4A+2\sin 5A\cos 2A+\sin 5A}{2\cos 5A\cos 4A+2\cos 5A\cos 2A+\cos 5A}$$

$$=\frac{\sin 5A\ (2\cos 4A+2\cos 2A+1)}{\cos 5A\ (2\cos 4A+2\cos 2A+1)}=\frac{\sin 5A}{\cos 5A}.$$

and

$$\frac{\sin A+\sin 2A+\sin 3A+\sin 4A+\sin 5A}{\cos A+\cos 2A+\cos 3A+\cos 4A+\cos 5A}$$

$$=\frac{(\sin A+\sin 5A)+(\sin 2A+\sin 4A)+\sin 3A}{(\cos A+\cos 5A)+(\cos 2A+\cos 4A)+\cos 3A}$$

$$=\frac{2\sin 3A\cos 2A+2\sin 3A\cos A+\sin 3A}{2\cos 3A\cos 2A+2\cos 3A\cos A+\cos 3A}$$

$$=\frac{\sin 3A\ (2\cos 2A+2\cos A+1)}{\cos 3A\ (2\cos 2A+2\cos A+1)}=\frac{\sin 3A}{\cos 3A}.$$

$$\therefore \text{Sum}=\frac{\sin 5A}{\cos 5A}+\frac{\sin 3A}{\cos 3A}$$

$$=\frac{\sin 5A\cos 3A+\sin 3A\cos 5A}{\cos 5A\cos 3A}$$

$$=\frac{\sin(3A+5A)}{\cos 5A\cos 3A}=\frac{\sin 8A}{\sin 3A\sin 5A}.$$

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PROBLEMS.

118. To find the local time at Ottawa on 1st February, 1880, when the sun will pass behind a tower which stands 30° south of the west point.

119. To find the probability of throwing n rings whose diameters are (1) in geometrical progression, (2) in arithmetical progression, over an upright standard. It may be assumed that the rings are thrown so that their planes are perpendicular to their direction of motion, or nearly so, and that a circle, radius a, may be drawn, having for centre the apex of the standard to represent unity; so that a will depend inversely on the skill of the thrower.

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120. A square, 160 feet side, has four circular flower beds, the centres of which are at the angular points of square, and radii 80 feet; find the area of the square unoccupied by the flower beds, and the radius of the smaller circle which will touch the four circles, given $\sqrt{2}=1.4142$ (approx.)

121. Prove that

$$3(1+1)+\frac{5}{\underline{|1}}\left\{(n+1)+2n+(n-1)\right\}+$$

$$\frac{7}{\underline{|2}}\left\{(n+2)(n+1)+3(n+1)n\right.$$
$$\left.+3n(n-1)+(n-1)(n-2)\right\}$$
$$+\frac{9}{\underline{|3}}\left\{(n+3)(n+2)(n+1)+4(n+2)(n+1)n\right.$$
$$+6(n+1)n(n-1)+4n(n-1)(n-2)$$
$$\left.+(n-1)(n-2)(n-3)\right\}+\ldots=0,$$

where n is positive and $\not> 2$.

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122. If $\frac{A+B+C}{abc}=\frac{A}{a}+\frac{B}{b}+\frac{C}{c}$ (1)

and $(A+B+C)(a+b+c)=Aa+Bb+Cc$, (2)

then will $\frac{A}{1+a^2}+\frac{B}{1+b^2}+\frac{C}{1+c^2}=0$

and also $\frac{A}{a+\frac{1}{a}}+\frac{B}{b+\frac{1}{b}}+\frac{C}{c+\frac{1}{c}}=0.$

$$(A+B+C)\ (a+b+c)=Aa+Bb+Cc,$$

$$\left.\begin{array}{l}\therefore\ A(b+c)+B(a+c)+C(a+b)=0\\ \text{and from (1)}\\ A(1-bc)+B(1-ac)+C(1-ab)=0\end{array}\right\}$$

$$\therefore \frac{A}{(1-ac)(a+b)-(a+c)(1-ab)}$$
$$=\frac{B}{(b+c)(1-ab)-(1-bc)(a+b)}$$
$$=\frac{C}{(1-bc)(a+c)-(b-c)(1-bc)},$$