SIMCOE.—Meteors reported by other persons, but not seen by observer. Snow, 11th, 15th, 18th, 21st, 27th. Rain, 2nd, 8th. Winter set in 14th month very cold. Sky murky.

HAMILTON.—Snow, 1st, 2nd, 11th, 12th, 21st, 23rd—27th. Rain, 2nd, 8th, 23rd.

WINDSOR.—Meteors: one in E. towards N., 3rd; one in S. W. towards H., 12th; three between Auriga and Ursa Major, 13th. Lunar halo, 29th. Snow, 11th, 19th, 23rd, 25th, 26th. Rain, 2nd, 8th, 18th, 23rd.

VIII. Mathematical Department.

EXAMINATION OF PUBLIC SCHOOL TEACHERS FOR FIRST CLASS CERTIFICATES, DECEMBER, 1873.

(Solution of questions in Natural Philosophy, Algebra and Geometry)

NATURAL PHILOSOPHY.

1. Let S be the sp. gr. of the liquid, and s of the body. Then, since the weight of a cubic foot of water is 1,000 oz., the weight of a cub. ft. of the liquid is 1,000 S ounces, and the weight of a cubic ft. of the body is 1,000 s ounces. Let V be the volume of the body in cub. feet; and p the number of cub. ft. immersed, before any pressure is applied. Then, by the conditions of the question, we have

$$1000Sp=1000sV,$$
$$\text{and, } 1000S\,(V-p)=1000sV+m.$$
$$\therefore\ 1000\,SV=1000V(2s)+m$$
$$\therefore\ 1000V\,(S-2s)=m.$$
$$\text{But, by the question, } S-2s=\tfrac{1}{1000}.$$
$$\therefore\ V=m.$$

[This question, in which there is no difficulty, was solved by none of the candidates. G. P. Y.]

2. It is not necessary to answer the first two parts of this question, (a) and (b). With respect to the third part (c), we have $8=\frac{1}{2}f\ \therefore f$ is 16, and consequently one half of g.

3. Resolve the forces in the directions of AE and of a line at right angles to AE. We may take the side of the parallelogram ABCD as unity, in which case AE$=\frac{1}{2}$. The resolved parts in the direction of AE are,

$$\tfrac{1}{2},\ -\tfrac{1}{2}-\frac{\sqrt{3}}{3},\ \tfrac{1}{4}+\frac{\sqrt{3}}{12};$$

and the sum of these is $\frac{1}{4}(1-\sqrt{3})$. The resolved parts in the direction at right angles to AE are,

$$\frac{\sqrt{3}}{2},\ 0,\ -\tfrac{1}{4}-\frac{\sqrt{3}}{4}$$

The sum of these is $-\frac{1}{4}(1-\sqrt{3})$. It thus appears that the given forces are equivalent to two, which are of the same absolute magnitude, one acting in the direction of AC, and the other in a direction at right angles to AC. The resultant of these must necessarily be in the direction of the diagonal of a square described on AC.

4. Let T be the tension of the string, R the reaction at C, and W the weight of the rod. Also let the normal to the plane at C meet AB in F; and let G be the middle point of AC. In order that there may be equilibrium, the directions of the three forces acting on the rod must pass through the same point. Therefore FG is at right angles to AC. Therefore (4 I. E.) AF=FC. But the resolved parts, in a horizontal direction, if T and R must counterbalance one another. That is,

$$T\tfrac{AG}{AF}=R\tfrac{CG}{CF}.$$
$$\text{But } AG=GC, \text{ and } AF=FC\ \therefore\ T=R.$$

5. Let C be the centre of gravity of the two particles. Then $\frac{n}{m}=\frac{AC}{BC}$ Suppose that, in a time t, the one particle has moved from A to D, and the other from B to E. Then,

$$AD=\frac{nft^2}{2(m+n)},\quad \text{and } BE=\frac{mft^2}{2(m+n)}.\ \text{Therefore}$$
$$\frac{AD}{BE}=\frac{n}{m}=\frac{AC}{BC}=\frac{AC-AD}{BC-BE}=\frac{DC}{EC}.$$

This conclusion, that $\frac{DC}{EC}=\frac{n}{m}$, means that the centre of gravity of the two particles has not altered its position.

6. The portions of the rope, BC and CD, weighing 4 lbs. and 3 lbs. respectively, may be supposed to be collected at the middle points of BC and CD. But, by the law of the inclined plane, a weight of 4 lbs. on BC exactly balances a weight of 3 lbs. on CD. Therefore, since the portion of the rope DE weighs 1 lb. more than the portion BA, a weight of 1 lb. must be attached to A in order that equilibrium may subsist.

7. Since the inclinations of the planes AB and BC are equal, P, which weighs 32 lbs., will counteract 32 lbs. of the weight of W, leaving only 8 lbs. as the weight by which motion is produced. But the weight to be moved is 9 times as much as this; because 40 lbs. are moved along BC, and 32 lbs. along AB, which (since the inclinations of AB and BC are equal) is the same thing as 72 lbs. along BC. Hence the acceleration will only be one-ninth part of what it would be in the case of a body falling down BC by its own weight. Therefore (if f be the acceleration of W descending down BC and dragging P up AB),

$$f=\frac{g}{9}\times\frac{BD}{BC}=\frac{32}{9}\times\frac{9}{16}=2.$$

Put $f=2$, and $t=1$, in the formula.

$$s=\tfrac{1}{2}ft^2,$$

and we get $s=1$.

ALGEBRA.

1. Let the rates at which the clocks A, B and C, go, be in the proportion of x^2, x, and 1. Then, when C indicates midnight, B has gone $12x$ hours, and A has gone $12x^2$ hours. Therefore, by the question,

$$12\times 60\,(x^2-x)=2\tfrac{1}{180}$$
$$\therefore\ x=\tfrac{361}{360}$$

Now A's rate exceeds B's (which is the true rate) in the proportion of x to 1, that is, in the proportion of 3610 to 3600. Hence in 3600 seconds, or one hour, true time, A gains 10 seconds. The loss of C (in the last line of the question, by an obvious misprint, B is written for C) is in like manner found to be $9\frac{351}{361}$.

2 [Mr. Jeffers was the only candidate who solved this question correctly. The following solution will, perhaps, be more easily apprehended than that given by Mr. Jeffers.—G. P. Y.]

Let x=the number of leaps by which the hare is ahead of the greyhound.

$2y$=the length of each of the hare's leaps.
$3y$= - - - - - - - greyhound's leaps.

$\frac{12}{m}$= number of hare's leaps per second.

$\frac{9}{m}$=number of greyhound's leaps per second.

$2xy$=original distance between gr. and h.

From these data, it is obvious, that, in one second, the greyhound gains $\frac{3y}{m}$ on the hare. Therefore it takes $\frac{mx}{3}$ seconds to gain xy, or to reduce its distance from the hare to one-half what it was originally.

But, at this point, when the distance of gr. from h. is only xy, the gr. increases its number of leaps per second to $\frac{9n+m}{mn}$. Therefore, in the remaining part of the course, its gain on the hare per second is $\frac{3y\,(m+n)}{mn}$. Hence, it gains xy, or catches the hare, in $\frac{xmn}{3\,(m+n)}$ seconds. This (by the question) is less by t seconds than $\frac{mx}{3}$, the number of seconds in which, at its original speed, it would have gained xy on the hare. That is,

$$\frac{mx}{3}-\frac{mnx}{3(m+n)}=t.$$
$$\therefore\ x=\frac{3t\,(m+n)}{m^2}.$$

3. The first part of this question is too easy to need to be solved here. It was solved by most of the candidates; none of them were right in the second part. When a is zero, the theorem does *not* hold good. For instance, the equations

$$(m+n)\,pq=p+q,$$
$$(p+q)\,mn=m+n,$$

would, if a were zero, be satisfied by the values, $p=1$, $q=-1$; for, when $a=0$, $m+n=0$; so that the expressions on both sides of each of the equations would vanish. And yet 1 and -1 are not roots of the equation,

$$3x^2+2=0.$$

How comes it to pass that the theorem fails in this particular case; I leave this question for the consideration of students. A candidate for a First Class Certificate ought certainly to be able to explain,