

$P$  in  $S$  cuts the curve  $S'$  at a constant angle in  $F'$ , tangent to evolute of  $S$  makes with evolute of  $S'$  a constant angle.

6. If  $S$  and  $S'$  intersect in the point  $O$ , the arc  $OP'$  bears a constant ratio to the difference between the arc  $OP$  and the tangent  $PP'$ .

The logarithmic spiral will serve to illustrate the two last theorems.

7. If right lines drawn from any point  $R$  in the curve  $S$  to touch the curves  $S'$  and  $S''$  in the points  $P$  and  $Q$  are equal, the product of the tangents of the halves of the angles which the lines  $RP$ ,  $RQ$  make with the tangent to  $S$  at the point  $R$  is constant.

As particular examples of this theorem we may take, firstly, the case of tangents drawn to a circle from any point in a line given in position.

Secondly, tangents drawn to two given circles from any point in their radical axis.

8. In the same figure as the last, if instead of having the tangents equal we have the angle  $PRQ$  constant, the circle passing through the three points  $P$ ,  $R$ ,  $Q$ , touches the curve  $S$  at the point  $R$ , and the normals to the three curves at the points  $P$ ,  $R$ ,  $Q$ , meet in a point.

9. If right lines drawn from any point  $R$  in the curve  $S$  touching the curve  $S'$  in the points  $P$  and  $Q$  contain with the arc  $PQ$  a constant area, tangent at  $R$  is parallel to the right line joining  $P$  and  $Q$ .

10. If the vertex of a constant angle is at the point  $O$ , and the sides of the angle cut the curve  $S$  in the points  $P$  and  $Q$ , and the curve  $S'$  in  $P'$  and  $Q'$ , area of the figure  $PQ P'Q'$  is a maximum when difference of squares of  $OP$  and  $OP'$  is equal the difference of squares of  $OQ$  and  $OQ'$ .

Hence if from a point  $O$  outside a circle it is required to draw two secants containing a given angle, so that the area of the figure contained by the secants and the circumference of the circle may be a maximum, it is w<sup>l</sup> the secants make equal angles with the diameter passing through the point  $O$ .

11. If the vertex of a constant angle is at the point  $O$ , and the sides of the angle cut the curve  $S$  in  $P$  and  $Q$ , the sum of  $OP$  and  $OQ$  is a minimum when the ratio of  $OP$  to  $OQ$  is equal to the ratio of the tangents of the angles which the sides of the given angle make with the curve.