(II) UNITED STATES METHOD.

Station.	B.S.	H. of I.	F.S.	Grade.	Grade rod.	Left.		Centre.	Right.		
102...	...	311.20	...	293.0	18.2	(s.s.) $\frac{+8.0}{22.0}$	$\frac{+8.5}{3.0}$	$\frac{+8.0}{0.0}$	$\frac{+7.5}{2.0}$	$+\frac{6.0}{8.0}$	$+\frac{7.0}{20.0}$ (s.s.)

The greater simplicity and convenience of the second method commend it to general use in spite of its drawbacks.

On some roads it is customary to take sections after the work is done, and pay for the actual quantities excavated; in others, the slope lines are adhered to and every endeavor is toward the full section taken out. It is probable, however, that the former method is more satisfactory to all concerned, although giving a little extra work to the engineering staff.

Structures need to be staked out twice, once for the foundation pit and again for the laying out of masonry, and of course, on all important structures, measurements and levels are given very frequently as the work progresses, both as an aid and a check to the contractors. A separate note book called a "structure book" should be used, and in it recorded, from day to day, notes of actual sizes, heights and measurements of all structures, and a duplicate office copy also kept up to date for fear of losing the field book. This structure book should include notes on all timber, stone and iron structural work, and will be a valuable aid in case of disputes involving quantities, it will also enable a large scaled profile being made for the maintenance department showing the exact chainage and elevation of each foundation and portion of structure. The necessity for absolute accuracy in laying out, measuring and calculating the quantities in all structures and grading cannot be too firmly impressed on the young engineer; all office calculations should be done in duplicate and preferably by different persons, as one cannot be very sure of a check on one's own work.

Cross Section Areas may be calculated in at least three or four ways, (1) where only a three-level section is taken, as in Fig. 77, the area of the section is evidently made up of triangles and is:

$$\text{Area} = \left(d \times \frac{S_1 + S_2}{2}\right) + \left(\frac{b}{2} + \frac{d_1 + d_2}{2}\right) \quad \text{........(1)}$$

(2) On rougher ground where more than three surface readings are necessary this method fails and must be replaced by more tedious ones, quite ordinarily Fig. 78 illustrates the one adopted, and consists in taking out the sum of all the trapezoidal areas, $A.A._1 D._1 D.E.F.G.H.A.$ and deducting the area of the two triangles $A.A._1 B.$ and $D.D._1 C.$

(3) By careful plotting, irregular areas can be taken out quite accurately with a polar planimeter.

(4) By Eckel's formula, which can be used without plotting the sections, and is equally adapted to the easiest or most difficult rectilinear areas. This formula, which is mathematically correct, is: "If the corners of any rectilinear polygon be referenced by rectangular co ordinates to any origin, then if the ordinate of every corner be multiplied by the abscissa of the next corner, and so on around the polygon, and these products added together; and if the ordinate of every corner be again taken and multiplied by the abscissa of the next corner, passing around the polygon in the *reverse* direction, and these products added together, then the area of the polygon is equal to one-half of the difference of these two sums." As an example, in Fig. 79 the area of the polygon is

$$\tfrac{1}{2}\{[(a \times d) + (c \times f) + (-ex - h) + -g \times -l) + (k \times b)] - [(a \times -l) + (k \times -h) + (-g \times f) + (-c \times d) + (c \times b)]\} \quad \text{........(2)}$$

or,

$$\tfrac{1}{2}\{(a \times d) + (c \times f) + (c \times h) + (g \times l) + (k \times b) + (a \times l) + (k \times h) + (g \times f + (c \times d) - (c \times b)\} \quad \text{........(3)}$$

In which great care must be taken to use the correct plus and minus signs. In railway work, this is much simplified by having all the area above the axis $XX.$, and in very irregular areas, which are met with in cuts that have slipped in as in Fig. 79, the area can be quickly taken out, thus, as follows:—

$$\text{Section} = \frac{+8.0}{-22.0}.\ \frac{+8.5}{-3.0}.\ \frac{+8.0}{0.0}.\ \frac{+7.5}{+2.0}.\ \frac{+6.0}{+8.0}.\ \frac{+7.0}{+20.0}.$$

$$\frac{+3.0}{+13.0}\ \frac{+0.0}{+9.0}.\ \frac{+0.0}{0.0}.\ \frac{+0.0}{-9.0}.\ \frac{+2.0}{-14.0}.\ \frac{+4.0}{-16.0}.\ \frac{+8.0}{-22.0}$$

$$\text{Area} = \tfrac{1}{2}\{-24.0 + 16.0 + 60.0 + 120.0 + 91.0 + 27.0 - 32.0 - 88.0 + 128.0 + 56.0 + 18.0 - 60.0 - 56.0 - 12.0 + 24.0 + 187.0\} = 227.5 \text{ square feet.}$$

Thus arriving at a correct result without plotting sections, and by a mechanical sort of process, which is a safe one to place in the hands of a comparatively unintelligent rodman; for the purpose of checking calculations, it is not appreciably more or less rapid than by taking out areas by method. (2)

QUANTITIES.

The use of tables and diagrams is a great aid in taking out approximate quantities, so that in various handbooks may be found the volume of 100 foot prismoids of level sections, of various heights, slopes and widths of road-bed, and this has been extended in Wellington's earthwork diagrams, etc., by giving the volumes of 100 foot prismoids where the sections, although not level, are of the three-level type, having a separate height at the centre and each slope stake, and as in easy sections this is all that is taken the diagrams are very useful. More accurate calculations of volumes of excavation or embankment may be made in three ways: (1) The prismoidal formula, which is the only one that is mathematically correct, is as follows:—

$$\text{Volume} = L \times \frac{A + 4A_1 + A_2}{6} \quad \text{........(4)}$$

Where L = length of prismoid A and A_2 = end areas and A_1 = middle area (which must be calculated by interpolating the middle heights).

A proof of this formula may be found in any mathematical text book, but a neat adaptation of the formula for three-level sections is given by Mr. G. H. White in *Engineering News*, April, 1895 (see Fig. 80).

$$\text{Volume} = \tfrac{1}{6} L\left\{A + 4A_1 + A_2\right\} =$$

$$\tfrac{1}{6} L\left\{\left(\frac{W.d}{2}\right) + \left(\frac{b}{2}\cdot\frac{H+h}{2}\right) + \left(\frac{W_2.d_2}{2}\right) + \left(\frac{b}{2}\frac{H_2+h_2}{2}\right) + 4\left[\tfrac{1}{2}\left(\frac{W+W_2}{2}\cdot\frac{d+d_2}{2}\right) + \frac{b}{2}\frac{\left(\frac{H+H_2}{2}+\frac{h+h_2}{2}\right)}{2}\right]\right\} \quad \text{......(5)}$$

$$= \tfrac{1}{6} L\left\{\left(\frac{W.d}{2} + \frac{W.d}{2} + \frac{W.d_2}{2}\right) + \left(\frac{W_2 d_2}{2} + \frac{W_2 d}{2} + \frac{W_2 d_2}{2}\right) + \frac{b}{4}\left(H + H_2 + h + h_2 + 2H + 2H_2 + 2h + 2h_2\right)\right. \quad \text{..........(6)}$$

$$= \tfrac{1}{6} L\left\{W\left(d + \frac{d_2}{2}\right) + W_2\left(d_2 + \frac{d}{2}\right) + \tfrac{3}{4}b(H + H_2 + h + h_2)\right. \quad \text{..........(7)}$$

and if we have a definite slope which we call s. (say $1\frac{1}{2}$ to 1 for earth, or $\frac{1}{4}$ to 1 for rock), we will have

$$H + H_2 + h + h_2 = W + \frac{W_2 - 2b}{s} \quad \text{..........(8)}$$

and the volume equation becomes

$$= \tfrac{1}{6} L\left\{W\left(d + \frac{d_2}{2}\right) + W_2\left(d_2 + \frac{d}{2}\right) + \tfrac{3}{4}\frac{b}{s}(W + W_2 - 2b)\right. \quad \text{..........(9)}$$