

With $T = 137.5$ seconds, $T_0 = 194.5$ seconds, $T_1 = 147$ seconds, $h_1 = 9.6$ feet, we get the following table:

for $T =$	10 sec.	100 sec.	200 sec.
$b_1 = h_1 \left[\frac{T_0}{T} \left(1 - \frac{T^2}{T_0^2} \right) - 1 \right] =$	+83.3 feet	-.264 feet	-4.93 feet
$b_2 = \frac{h_1}{T} =$	.96 $\frac{\text{feet}}{\text{sec.}}$	.096 $\frac{\text{feet}}{\text{sec.}}$	.048 $\frac{\text{feet}}{\text{sec.}}$
$R = h_1 \cdot \frac{T_0}{T} \cdot \frac{T_1}{T} =$	200 feet	20 feet	10 feet
$tg\beta = \frac{2 \left(1 - \frac{T^2}{T_0^2} \right) T_0}{\left(3 - \frac{T^2}{T_0^2} \right) T_1} =$	.530	.530	.530
	$\beta = 207^\circ 54'$	$\text{arc } \beta = 3.628$	$\gamma = 69^\circ 18'; \text{ arc } \gamma = 1.210$
$z_T =$	-9.10 feet	-4.85 feet	-1.05 feet
$s_T =$	.097 $\frac{\text{feet}}{\text{sec.}}$	.091 $\frac{\text{feet}}{\text{sec.}}$	.0735 $\frac{\text{feet}}{\text{sec.}}$
$R_1 \sin \beta_1 =$	-9.10 feet	-4.85 feet	-1.05 feet
$R_1 \cos \beta_1 =$	+11.05 feet	+11.6 feet	+10.4 feet
$R_1 =$	14.30 feet	12.55 feet	10.45 feet
$\beta_1 \text{ negative} =$	-39° 30'	-22° 44'	-5° 46'
$\text{arc } \beta_1 =$	-.6894	-.3968	-.1006
$z \text{ max} =$	+6.53 feet	+6.40 feet	+5.97 feet

From the graphical demonstration (Fig. 4)* we see that the period of shut-down up to 100 seconds influences the maximum elevation a very small amount indeed. The explanation for this is that during this period the velocity of flow in the main conduit decreases very little. For the determination of the dimensions of the surge tank, we have, therefore, to consider the results of the limiting case, that is, the sudden shut-down. The computation for that condition is a much simpler one, (case A). For a gradual opening, the same relations exist. The lowering of the level $n-n$ occurs to the same extent and for the same duration as the rise above the initial level in the case already computed.

Of special interest is the case of an outflow, which is variable in the sense that the outflow increases considerably during a certain time and decreases afterwards to the same amount as before or to some other amount, (for instance, in a plant for railway operation).

1. ANALYTICAL INVESTIGATION.—We assume now, that under circumstances similar to those above mentioned, the following law of outflow is effective:

$$q = \epsilon \cdot Q_1 (1 + f \sin t/T)$$

so that for the time

$$t = 0; \quad t = \frac{\pi \cdot T}{2}; \quad t = \pi T \quad q \text{ becomes resp. } = \epsilon Q_1;$$

$$\epsilon Q_1 (1+f); \quad \epsilon Q_1 \quad - \quad - \quad (68)$$

after the time $t = \pi \cdot T$ the outflow may be constant again. ϵ and f are natural numbers. f is the proportion of

the maximum increase of the outflow to the normal outflow. So that

$$c = \frac{q}{A} = \epsilon c_1 \left(1 + f \sin \frac{t}{T} \right) \quad \frac{dc}{dt} = \epsilon \cdot f \cdot \frac{c_1}{T} \cos \frac{t}{T}$$

Equation 23 may then be written:

$$\frac{d^2 z}{dt^2} + \frac{1}{T_0} \frac{dz}{dt} + \frac{z}{T^2} + \frac{\epsilon \cdot c_1}{T_0} + \frac{\epsilon \cdot f \cdot c_1}{T_0} \sin \frac{t}{T} + \frac{\epsilon \cdot f \cdot c_1}{T} \cos \frac{t}{T} = 0 \quad (69)$$

If we introduce

$$z = y - \frac{\epsilon \cdot c_1 T^2}{T_0} = y - \epsilon h_1; \quad \frac{dz}{dt} = \frac{dy}{dt}; \quad \frac{d^2 z}{dt^2} = \frac{d^2 y}{dt^2} \quad (70)$$

and further,

$$\epsilon \cdot f \cdot c_1 \left[\frac{1}{T_0} \sin \frac{t}{T} + \frac{1}{T} \cos \frac{t}{T} \right] = \epsilon \cdot f \cdot c_1 \sqrt{\frac{1}{T_0^2} + \frac{1}{T^2}} \sin \left(\phi + \frac{t}{T} \right) \quad (71)$$

with $tg \phi = \frac{1}{T}$ then the equation 69 transposes to

$$\frac{d^2 y}{dt^2} + \frac{1}{T_0} \frac{dy}{dt} + \frac{y}{T^2} + \frac{\epsilon \cdot f \cdot c_1}{T_0} \sqrt{\frac{1}{T_0^2} + \frac{1}{T^2}} \sin \left(\phi + \frac{t}{T} \right) = 0 \quad (72)$$

*See The Canadian Engineer August 27th, Page 370.