habit of observing these conditions only by practice in their application, and as affording such practice, mathematics stand pre-eminent. They constitute the most perfect application of pure logic, and especially of the deductive method of ascertaining truth. This method of deduction—of which Mathematics give the most scientific form—is a most powerful instrument in the discovery of truth. The laws of extension and number—that is, the laws of Mathematics—underlie all the other laws of the material universe; and in the great inquiries of the moral and social sciences, Mathematics afford the only sufficiently perfect type. "Up to this time," says Mill, "I venture to say that no one ever knew what deduction is as a means of investigating the laws of nature, who had not learned it from Mathematics, nor can any one hope to understand it thoroughly who has not, at some time of his life, known enough of Mathematics to be familiar with the instrument at work." The logical definition of terms—the explicit statement of premises—the clear and well-defined steps in the trains of reasoning—the exclusion of intermediate propositions the truth of which is not clearly seen—the precise and constant meanings of the terms employed—these are characteristics of the deductive method, which finds its highest type in Mathematics.

But it has been said that the Logic of Mathematics, dealing only with necessary matter, and concerned only with demonstrative evidence, does not prepare the mind for researches in contingent matter, that is, to correctly estimate probable evidence. It may be remarked in passing, that this objection is equally valid against Metaphysics and indeed all the rational sciences, which demand not probable but certain evidence.

But the objection has no weight when urged against Mathematical logic. For, to say nothing of the fact that the Mathematical theory of probabilities is a most valuable contribution to logic, and lays the foundation for a sound knowledge of the rules of probable evidence, what is to be our guide in estimating probable evidence, but those logical forms of reasoning deduced from the laws of mind, and practically exhibited in their highest perfection in Mathematics? Is pure logic of great value as pointing out the conditions of cogency to which the *probable* argument must conform in order to secure the acceptance of its conclusion as an article of belief? How much greater the value of Mathematics, which demand a continued application of these conditions, and hence educate the mind to a sagacity in detecting error, that the mere study of formal logic cannot impart? When we consider the multiplicity of circumstances likely to invalidate our investigations in the "field of probabilities," we can scarcely think too highly of those methods of discipline which develope so acute a perception of the form and essence of sound reasoning, that the mind is enabled instinctively, as it were, to detect the presence of fallacies. Mathematics proceed from data which have the certainty of necessary truths by a demonstrative process in which the connection between the successive steps of the reasoning is clearly comprehended. No obscure terms, nor imperfectly understood propositions, are either admitted as data or mark their processes. Should not a similar rigor be observed in reasoning on contingent matter? They assume no principle as the basis of an argument, or as a means of effecting a synthesis, whose truth has not already been established, and they submit to the severest test everything having the slightest element of uncertainty. Should not the same method distinguish inquiries in which there is a balancing of probabilities? The highest ingenuity and skill in analysis and combination, are required in Mathematical research—surely the same qualities are essential to correct reasoning in matters of observation and experience.

It has been also said that the *matter* and the *method* of Mathematics preclude the possibility of error, and that therefore the science does not, like probable reasoning, educate to sagacity in its detection. But, as already suggested, it is impossible to discover the fallacies of probable reasoning without practical skill in the methods of sound reasoning; and this, the study of Mathematics imparts by rigorous adherence to the forms of strict logical inference. "Let us be assured," says the great thinker already quoted, "that for the formation of a well-trained intellect, it is no slight recommendation of a study, that it is the means by which the mind is earliest and most easily brought to maintain within itself a standard of complete proof." It is true that Mathematics have continued to make unerring progress, while contradiction and abberration have distinguished most other sciences and retarded their development. This progress, however, does not prove the impossibility of mental sophistries in Mathematical investigation, but rather that the *matter* and the *method* of the science lead quickly to the detection of fallacies, and prevent the introduction of permanent error. Such fallacies are probably due to the abstract and comprehensive nature of the conceptions involved in the demonstrations; and that their discovery and elimination often require great skill, as well as acuteness and soundness of judgment is well known to every student of the science. If a vigorous exercise of intellectual power is necessary to grasp such conceptions, a still higher degree of mental energy is required to comprehend their *relations*; and thus the mind is led sometimes to confound abstractions which are really distinct; at others, to assume an analogy where none exists. Hence, if fallacies creep, into Mathematical demonstrations *in spite of the logical rigor of their method*, they must be such as are most likely to deceive the mind, and their frequent occurrence—with their discovery and correction—must habituate the student to a discriminating caution which is of great value in the probable reasonings of experience. A mind thoroughly trained in Mathematical reasoning may indeed commit the error of expecting in all proof too close an adherence to the type with which it is familiar; "but he who has never acquired this type has no just sense of the difference between what is proved and what is not proved; the first foundation of the scientific habit of mind has not been laid."

2. But further. *The study of Mathematics requires the exercise of ingenuity, acuteness in discrimination, and caution in the admission and combination of data*, and consequently affords a still more effective preparation for conquering the difficulties and avoiding the dangers in the reasonings of experience. Though Mathematical science is demonstrative—occupied with the deduction of conclusions—we are as often required to establish certain truths to serve as premises for the deduction of a proposed truth, as to deduce the necessary consequences from given premises. These premises are to be selected from the numerous truths already acquired by the understanding, and combined to effect the required proof: and this can be accomplished only by a careful analysis of the proposition to be proved, and an accurate discrimination of the results previously known. A careful examination of the given proposition is needed to guide the mind to the necessary data; accurate discrimination and ingenuity, to select, from the many principles bearing on the question, those necessary and sufficient for the demonstration.

In the solution of Mathematical problems, how is the synthesis between the known and the unknown to be effected, without skilful analysis, acute comparison, and judicious application? The relative bearings of principles previously determined, and their connection with those to be established, must be carefully examined and clearly comprehended as a preliminary to the required solution—does this require no acuteness in comparison and discrimination? The intermediate terms employed in the investigation must be sought among general truths which from their complex relations, are the more difficult to distinguish—is there no ingenuity required in the selection and application of those which will lead most directly to the desired result? The very fact of mathematics being a demonstrative yet a *progressive* science, proves at once the necessary *connection* yet *distinctiveness* of its propositions, and implies as a condition of progress the constant exercise of the powers in question. Hence as sagacity and skill are required in common reasoning to obtain the needed premises, and ingenuity in analysis and comparison to free them from everything irrelevant to the argument, it seems evident that mathematics must prepare us for overcoming the difficulties by which such reasoning is characterized, and for moulding the isolated facts furnished by observation and experience into the symmetry and stability of science. This seems to receive corroboration from the great success which mathematicians have achieved in the application of the science to external phenomena. For most of the physical sciences are founded on observation and experiment necessarily carried on by mathematicians, and eminently exhibiting subtlety in discrimination and analysis, and skill in comparison and generalization. It is a fair inference, too that mathematics qualify the mind for observation and experiment, since these sciences owe their origin to mathematical skill in observing and generalizing physical facts, as well as their development to the power of mathematical analysis.

But Mathematics induce a cautiousness in the admission and combination of data which still further fortifies the mind against the fallacies that occur in reasoning on practical affairs. However opposed to the progress of truth violation of the forms of true reasoning may be, a source of error equally fertile is to be found in the rash assumption of false premises, and

(CONTINUED PAGE 16.)