UNIVERSITY WORK.

MATHEMATICS.

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SOLUTION AND PROBLEMS.

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SOLUTION.

Given the sum of the sides, the base and the height of a triangle; construct it.

Let AB be the base. From centre A, and at a distance equal to the sum of the sides, describe a circle HKG. Draw AF at right angles to AB, and equal to the given altitude. Draw FD parallel to AB. Draw BD perpendicular to AB, and produce it making $BD = DE$. Describe a circle passing through B and E, and touching the circle HKG internally at K. Join AK, cutting FD at C; join CB and CE. Then the centre of this circle must be in DC, and also must be in AK; therefore C is centre, and ACB is the triangle required.

PROBLEMS.

Sum the following series to n terms:

(1) $4+17+54+145+368+$ etc.

(2) $4-14+117-632+3913-$ etc.

(3) $3-1+27-41+179-$ etc.

(4) $2+12+36+80+150+$ etc.

(5) $-4+3+22+59+120+$ etc.

(6) $3+13+25+41+65+$ etc.

(7) Describe a circle through two given points to touch a given circle.—*Selected.*

PROBLEMS.

1. In 11 prop. Bk. ii., show that the squares on the whole line, and on the less segment, are together equal to three times the square on the greater segment.

2. A circle is described, passing through the right angle of a right-angled triangle, and touching the hypotenuse at its middle point; prove that the area of this circle, intercepted within the triangle, is trisected at the middle point of the hypotenuse.

3. If a line be bisected and produced to any point, the square on the whole line thus produced is equal to the square on the part produced together with twice the rectangle contained by the line and the line made up of the half and the part produced.

4. In iv 10, Euclid. If in the construction AB be the line which is divided in C, and ABD be the triangle, and the two circles in the figure intersected aga in E, prove that CE is parallel to BD.

5. If the circle inscribed in the triangle ABC touch the sides at the points DEF respectively, and P be the point of concurrence of the lines AD, BE, CF, show that

$$\frac{PD}{AD}+\frac{PE}{BE}+\frac{PF}{CF}=1$$

with corresponding properties for the escribed circles.

6. If $\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)\left(\frac{b}{c}+\frac{c}{b}\right)+4=0$,

where α, β are the roots of $ax^2+bx+c=0$, show that $\alpha=\beta=2$.

7. If $A+B+C=180$

$$(y-z)\cot\frac{A}{2}+(z-x)\cot\frac{B}{2}+(x-y)\cot\frac{C}{2}=0,$$

$$(y^2-z^2)\cot A+(z^2-x^2)\cot B+(x^2-y^2)\cot C=0,$$

prove that

$$\frac{y^2+z^2-2yz\cos A}{\sin^2 A}=\frac{z^2+x^2-2zx\cos B}{\sin^2 B}=\frac{x^2+y^2-2xy\cos C}{\sin^2 C}.$$

8. Prove in any way that

$$a^2(b-c)(b+c-a)^2+b^2(c-a)(c+a-b)^2+c^2(a-b)(a+b-c)^2$$
$$=(a-b)(b-c)(c-a)\cdot[2(ab+bc+ca)-a^2-b^2-c^2].$$