$$\left(\frac{1}{s_1}+\frac{1}{s_2}+\frac{1}{s_3}+\dots\right)^2-\left(\frac{1}{s_2}+\frac{1}{s_4}+\frac{1}{s_6}+\dots\right)^2 = \frac{a^2}{a^2-1}\,\frac{(1-r)^2}{a^2-r^2}.$$

Show by an *ex absurdo* proof that a quadratic equation cannot have more than two roots.

Solve the following equations:

(i.) $\dfrac{50x^2+75x-1250}{5x+8}-\dfrac{40x^3-592x}{4x^2-7}+1=0.$

(ii.) $ax^2(y-z)=by^2(z-x)=cz^2(x-y)=d^4.$

Find the roots of the equation

$$9^x-8.3^x+3=0$$

to five places of decimals.

5. Assuming the binomial theorem to have been proved true for any positive integral exponent, prove its truth for any positive exponent.

If $\phi(x,n)=\dfrac{1}{x}-n\dfrac{1}{x+1}+\dfrac{n(n-1)}{1.2}\,\dfrac{1}{x+2}-\dots$

where n is a positive integer, find a relation connecting $\phi(x,n)$ and $\phi(x+1,\ n+1)$; and thence show that $\phi(x,n)=\dfrac{n!\,(x-1)!}{(x+n)!}$.

6. Assuming the expansion of e^x in ascending powers of x, deduce the expansion of $\log(1+x)$.

Hence, from the identity $x^3+1=(x+1)(x^2-x+1)$, show that, if m be a positive integer,

$$1-\frac{6m-2}{1.2}+\frac{(6m-3)(6m-4)}{1.2.3}-\frac{(6m-4)(6m-5)(6m-6)}{1.2.3.4}+\dots=0.$$

vii. Explain the different methods of measuring angles.

Find the number of degrees in each angle of a regular polygon of n sides, (1) when it is convex; (2) when its periphery surrounds the inscribed circle m times.

Find, correct to .01 of an inch, the length of the periphery of a decagon which surrounds an inscribed circle of a foot radius three times.

viii. Prove geometrically the formula

$$\cos\alpha+\cos\beta=2\cos\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta).$$

Prove that

$$\begin{aligned}&2\cos(\alpha-\beta)\cos(\theta+\alpha)\cos(\theta+\beta)\\&+2\cos(\beta-\gamma)\cos(\theta+\beta)\cos(\theta+\gamma)\\&+2\cos(\gamma-\alpha)\cos(\theta+\gamma)\cos(\theta+\alpha)\\&-\cos 2(\theta+\alpha)-\cos 2(\theta+\beta)-\cos 2(\theta+\gamma)-1\end{aligned}$$

is independent of θ, and exhibit its value as the product of cosines.

ix. Prove geometrically the formula

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}.$$

Prove that, if $\alpha, \beta, \gamma, \delta$ be four solutions of the equation $\tan(\theta+\frac{1}{4}\pi)=3\tan 3\theta$, no two of which have equal tangents, then

$$\tan\alpha+\tan\beta+\tan\gamma+\tan\delta=0,$$

and $\tan 2\alpha+\tan 2\beta+\tan 2\gamma+\tan 2\delta=\frac{4}{3}.$

x. Show that in general the change in the cosine of an angle is approximately proportional to the change in the angle.

Prove that, if in measuring the three sides of a triangle, small errors x, y be made in two of them a, b, the error in the angle C will be

$$-\left(\frac{x}{a}\cot B+\frac{y}{b}\cot A\right),$$

and find the errors in the other angles.

xi. Show that in any triangle

$$a\cos B+b\cos A=c,$$

and deduce the formula

$$c^2=a^2+b^2-2ab\cos C.$$

Prove that, if O be the centre of the circumscribed circle of the triangle ABC, the sides of the triangle formed by the centres of the three circles BOC, COA, AOB, will be proportional to $\sin 2A : \sin 2B : \sin 2C$; and find the angles of the new triangle correct to one second, when the sides of the triangle ABC are in the ratio of 4 : 5 : 7.

xii. Find the radius of the inscribed circle of a triangle in terms of one side and the angles.

Prove that, if P be a point from which tangents to the three escribed circles of the triangle ABC are equal, the distance of P from the side BC will be

$$\tfrac{1}{2}(b+c)\sec\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C.$$