where, since we may arrange to have h_1, h_2, &c., as small as we please, we may understand *that h is a quantity* which we can arrange to have as small as we please. In like manner, if S_3 be the sum of the angles of the triangle LDM, we can get

$$2-S_3 = n\,(2-S_1) \backsim k; \ldots\ldots\ldots\ldots \quad (6)$$

k being a quantity which we can arrange to have as small as we please. Hence, from (5) and (6), we can order our construction so as to make the ratio, $2-S_2 : 2-S_3$, as nearly equal as we please to the ratio, $N : n$; the same means by which this is secured having the effect of rendering [see (3) and (4)] the ratio, LED : LMD, as nearly equal as we please to the ratio, $N : n$. Hence we can order our construction so as to make the two ratios,

LED : LMD,

and, $2-S_2 : 2-S_3$,

as nearly equal as we please. This is accomplished by the means above described, whatever be the length of the line FD. It may therefore be still accomplished, though FD be taken indefinitely small. But as FD is indefinitely diminished, the area of the triangle LFD, and therefore that of the triangle LBE is (Prop. III.) indefinitely diminished. Hence, as FD is indefinitely diminished, the ratio of the triangles LED and LBD ultimately becomes indefinitely near to a ratio of equality; the ratio of the triangles LDM and LCM also becoming, under the same circumstances, indefinitely near to a ratio of equality. Consequently, by taking FD small enough, the ratio, LBD : LCD, or, $A : a$, becomes indefinitely near to the ratio, LED : LMD. In like manner it can be proved, that, as FD becomes indefinitely small, the ratio, $2-S_2 : 2-S_3$, approximates indefinitely to the ratio, $2-S : 2-s$. Therefore the ratio, $A : a$, cannot differ *by any finite amount from the ratio, $2-S : 2-s$. That is,*

$$A : a = 2-S : 2-s.$$

Proposition VI.

If BGC and HCF (Fig. 13) be any two plane triangles, S being the sum of the angles of the former, and s the sum of the angles of the latter; then, reasoning on the hypothesis that the angles of a