that if $n_2+n=1$, $x^2+nax+n^2$ is a factor. Or assuming another factor, $x^2+max+a^2$, we obtain the statement $(x^2+nax+a^2)(x^2+max+a^2)=x^4-ax^3+a^2x^2-a^3x+a^4$. Multiplying out and equating coefficients

$$m+n=-1$$
$$2+mn=1$$
$$\therefore\quad mn=-1 \text{ and}$$
$$m+n=-1.$$

From these two equations $m+n$ can be found.

8. Solve the equations:

(1) $\sqrt{(2x^2+1)}+\sqrt{(2x^2+3)}=2(1-x)$.

(2) $\dfrac{ax+b}{ax-b}-\dfrac{bx}{ax+b}=\dfrac{ax}{ax-b}-\dfrac{(ax_2-2b)b}{a^2x_2-b^2}$.

(3) $x+y+z=3a+b+c$
$x+y+t=a+3b+c$
$x-z-t=a+b-c$
$y+z-t=3a-b-c$

8. (1) Square $4x^2+4+2\sqrt{(2x^2+1)(2x^2+3)}$
$=4+4x^2-8x$.

$$\therefore\quad 2\sqrt{(2x^2+1)(2x^2+3)}=-8x.$$
$$\therefore\quad \sqrt{(2x^2+1)(2x^2+3)}=-4x.$$

Square again $4x^4+8x^2+3=16x^2$.

$$\therefore\quad 4x^4-8x^2+3=0.$$
$$\therefore\quad (2x^2-1)(2x^2-3=0.$$
$$\therefore\quad x=\pm\sqrt{\tfrac{1}{2}} \text{ or } \pm\sqrt{\tfrac{3}{2}}$$

(2) Transposing

$$\frac{b}{ax-b}-\frac{x}{ax+b}=-\frac{(ax2-2b)b}{a^2x^2-bc}$$
$$\therefore\quad \frac{1}{ax-b}-\frac{bx}{ax+b}=\frac{2b-ax2}{a^2x^2-b^2}$$

Simplifying, $\therefore\ ax+b-ax^2+bx=2b-ax^2$,

$$\therefore\ ax+bx=b,$$
$$\therefore\ x=\frac{b}{a+b}.$$

(3) A simple simultaneous equation. The value of y can be readily found by subtracting (2) from (1) and substituting the resultant value for $z-t$ in (4). By this process

$$v+2a-2b=3a-b-c$$
$$\therefore\quad y=a+b-c.$$

Adding (1), (2) and (3) together there results

$$3x+2y=5a+5b+c, \text{ but}$$
$$2y=2a+2b-2c,\quad\therefore$$
$$3x=3a+3b+3c,$$
$$\therefore\quad x=a+b+c.$$

The values of z and t can now be readily found.

9. A grocer had three casks of wine containing in all 344 gallons. He sells 50 gallons from the first cask; then pours into the first one-third of what is in the second, and then into the second one-fifth of what is in the third, after which the first contains 10 gallons more than the second, and the second 10 more than the third. How much wine did each cask contain at first?

9. Let $x=$ number of gallons in 1st cask,
$y=$ " " 2nd "
$z=$ " " 3rd "

$$\therefore\quad x+y+z=344\quad(1)$$

also $x-50+\dfrac{y}{3}=10+\dfrac{2}{3}y+\dfrac{z}{5}=20+\dfrac{4}{5}z$ (2)

$$\therefore\ x-\frac{y}{3}-\frac{z}{5}=60\quad(3)$$

also $x+\dfrac{y}{3}-\dfrac{4}{5}z=70$ (4)

Subtracting (3) from (1) we have

$$\frac{4}{3}y+\frac{6}{5}z=284\quad(5)$$

also (3) from (4) $\dfrac{2}{3}y-\dfrac{3}{5}z=10$ (6)

from (5) and (6) we get $z=110$, $y=114$,
$\therefore$ from (1) $x=120$.

10. Given the sum of an Arithmetical Progression, the first term, and the common difference, find the number of terms (n).

(1) Interpret the result when there is a negative value of n.

10. Book-work.

(1) $s=\dfrac{n}{2}(2a+\overline{n-1}\,d)$. Let a value of n which will satisfy this $-n$ be $-m$,

$$\therefore\quad s=-\frac{m}{2}(2a+\overline{-m-1}\,d),$$
$$\therefore\quad 2s=-m(2a-\overline{m+1})d$$
$$=-2am+m(m+1)d$$
$$=m[2(d-a)+(m-1)d]$$
$$\therefore\quad s=\frac{m}{2}[2(d-a)+(m-1)d]$$

which is the sum of a series whose first term is $(d-a)$ and common difference d, and number of terms m.

11. (1) If n geometric means be found between p and q, determine their product.

(2) If x, y, z are in G. P. show that

$$x^2y^2z^2\left(\frac{1}{x^3}+\frac{1}{y^3}+\frac{1}{z^3}\right)=x^3+y^3+z^3.$$