The H. C. F. of the expression is binomial of one dimension in x. Operating on coefficients alone.

$$\begin{array}{rlrl} \text{A} & 1+p+g & \text{B} & 1+m+n \\ \text{B} & 1+m+n & g\text{B} & g+gm+gn \\ \text{A}-\text{B} & (p-m)+(g-n) & n\text{A} & n+np+ng \\ & & g\text{B}-n\text{A} & (g-n)+(gm-np) \\ \div p-m & 1+\dfrac{g-n}{p-m} & \div g-n & 1+\dfrac{gm-np}{g-n}. \end{array}$$

$$\therefore \frac{g-n}{p-m}=\frac{gm-pn}{g-n};$$

or $(p-m)(gm-pn)=(g-n)^2$ is the necessary relation. Or, as $(p-m)x+(g-n)$ and $(g-n)x+(gm-pn)$ can differ only by a monomial factor, we must have $p-m : g-n = g-n : gm-pn$. $\therefore$ etc.

9 (*a*). Solve $\left.\begin{array}{r} x^2-7xy+11y^2=179 \\ 2x-y=1 \end{array}\right\}$

$y=2x-1$, and substituting this value of y in the first equation gives $31x^2-37x=168$.

Whence $x=3$ or $-1\frac{25}{31}$; $y=5$ or $-4\frac{19}{31}$.

(*b*). Solve $\dfrac{x-b}{x-a}-\dfrac{x-a}{x-b}=\dfrac{2(a-b)}{x-a-b}$.

$$\frac{x-a+a-b}{x-a}-\frac{x-b+b-a}{x-b}=\frac{a-b}{x-a}+\frac{a-b}{x-b}=\frac{2(a-b)}{x-a-b}.$$

$$\therefore \frac{2x-a-b}{(x-a)(x-b)}=\frac{2}{x-(a+b)}.$$

Whence by clearing of denominators,
$2x^2-3(a+b)x+(a+b)^2=2x^2-2(a+b)x+2ab$.

$\therefore$ $x=\infty$, and $x=\dfrac{a^2+b^2}{a+b}$.

10. (*a*). Simplify $\dfrac{a^3}{(a-b)(a-c)}+\dfrac{b^3}{(b-c)(b-a)}+\dfrac{c^3}{(c-a)(c-b)}$.

Arrange the denominators in cyclic order, and put in the factor which makes each $(a-b)(b-c)(c-a)$.

The numerator becomes $a^3(c-b)+b^3(a-c)+c^3(b-a)$.

This factors into $(a-b)(b-c)(c-a)(a+b+c)$.

$\therefore$ The simplified expression is $a+b+c$.

(*b*). The area of a rectangle is to that of the square on its diagonal as 60 : 169. Find the ratio of the sides of the rectangle.

Let x, y be the sides of the rectangle. Area of the rectangle $=xy$, and the area of the square on its diagonal is x^2+y^2.

$\therefore$ $xy : x^2+y^2=60 : 169$, to find $\dfrac{x}{y}$

Divide the first couple by y^2. Then $x/_y\, 169=[(x/_y)^2+1]60$; and taking $x/_y$ as variable, and solving we get, $x/_y={}^5/_{12}$ or ${}^{12}/_5$.